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Continuous Probability Distribution

If a random variable takes any value in an interval, it will give rise to Continuous distribution

Continuous probability function: If for every x belongs to continuous random variable X, we assign a real number $f(x)$ satisfying the conditions

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x).dx = 1$

then $f(x)$ is called a continuous probability function (or) probability density function (Pdf)

If (a,b) is a subinterval of the range space of X then the probability that x lies in (a,b) is defined to be the integral of $f(x)$ between a and b.

i.e $P(a \leq x \leq b) = \int_a^b f(x).dx$

Cumulative distribution function

If X is a Continuous random variable with Pdf $f(x)$ then the function

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$ is called the cummulative distribution function of X.

If r is any real number then

1. $P(x \geq r) = \int_r^{\infty} f(x)\,dx$

2. $P(x < r) = 1 - P(x \geq r)$

$= 1 - \int_r^{\infty} f(x).dx$

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Mean & Variance

If X is a continuous random variable with probability density function $f(x)$ where $-\infty < x < \infty$, the mean(μ) or Expectation $E(X)$ defined and the variance (σ^2) of X is defined as

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\, dx.$$

$$\text{Variance } (\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \quad \text{or} \quad \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

~~Exponential Problems~~ Exponential Distribution

The continuous probability distribution having the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise, where } \alpha > 0 \end{cases}$$

is Known as the exponential distribution.

Mean and SD of the Exponential Distribution

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\, dx.$$

$$\mu = \int_{0}^{\infty} x \cdot \alpha e^{-\alpha x}\, dx = \alpha \int_{0}^{\infty} x e^{-\alpha x}\, dx$$

$$\mu = \alpha \left[x \cdot \frac{e^{-\alpha x}}{-\alpha} - 1 \cdot \frac{e^{-\alpha x}}{\alpha^2} \right]_0^{\infty} \qquad \left\{ \frac{x}{e^{\alpha x}} \to 0 \text{ as } x \to \infty \right.$$

$$\mu = \alpha \left[\frac{-1}{\alpha}(0-0) - \frac{1}{\alpha^2}(0-1) \right]$$

$$\therefore \mu = \frac{1}{\alpha}$$

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Variance $(\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x) \cdot dx \Rightarrow$

$$\sigma^2 = \alpha \int_0^{\infty} (x-\mu)^2 e^{-\alpha x} . dx.$$

$$\sigma^2 = \alpha \left[(x-\mu)^2 \left[\frac{e^{-\alpha x}}{-\alpha}\right] - 2(x-\mu)\left[\frac{e^{-\alpha x}}{\alpha^2}\right] + 2\left[\frac{e^{-\alpha x}}{-\alpha^3}\right]\right]_0^{\infty}$$

$$= \alpha \left[\frac{-1}{\alpha}\{0-\mu^2\} - \frac{2}{\alpha^2}\{0-(-\mu)\} + \frac{2}{\alpha^3}\{0-1\}\right]$$

$$= \alpha \left[\frac{\mu^2}{\alpha} - \frac{2\mu}{\alpha^2} + \frac{2}{\alpha^3}\right]$$

But $\mu = \frac{1}{\alpha}$.

$$\therefore \sigma^2 = \alpha\left[\frac{1}{\alpha^3} + \frac{\cancel{2}}{\cancel{\alpha^3}} + \frac{\cancel{2}}{\cancel{\alpha^3}}\right] = \frac{1}{\alpha^2}$$

$$\therefore \boxed{\text{S.D } (\sigma) = \frac{1}{\alpha} \quad \& \quad \text{mean } (\mu) = \frac{1}{\alpha}.}$$

$\Rightarrow$ Mean & standard deviation of exponential distribution are equal.

HW Find k such that $f(x) = \begin{cases} kxe^{-x}, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$ is a P.d.f. Find the mean

$K = \frac{e}{e-2}$

$\mu = \frac{2e-5}{e-2}$

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① Find the constant K $=1/9$ such that $f(x) = \begin{cases} Kx^2, & 0<x<3 \\ 0, & \text{otherwise} \end{cases}$ is a Pdf

Also compute (i) $P(1<x<2)$ $7/27$ (ii) $P(x \le 1) = \frac{1}{27}$

(iii) $P(x>1)$ $26/27$ (iv) Mean $9/4$ (v) Variance $27/80$

② A random variabe x has the density function $f(x) = \frac{K}{1+x^2}$, $-\infty < x < \infty$

Determine K and hence evaluate (i) $P(x \ge 0)$ (ii) $P(0<x<1)$

$K = \frac{1}{\pi}$ $\frac{1}{2}$ $\frac{1}{4}$

③ If x is an exponential variate with mean 5, evaluate

(i) $P(0<x<1)$ 0.1813 (ii) $P(-\infty<x<10)$ 0.8647 (iii) $P(x \le 0 \text{ or } x \ge 1)$ 0.8187

④ The sales per day in a shop is exponentially distributed with the average sale amounting to Rs 100 and net profit is 8%. Find the probability that the net profit exceeds Rs 30 on two consecutive days. $e^{-7.5} = 0.00055$

⑤ In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. what is the probability that a shower will last for:

(i) 10 minutes (or) more 0.1353 (ii) less than 10 minutes 0.8647

(iii) between 10 and 12 minutes 0.0446

⑥ The length of telephone conversation in a booth has been an exponential distributed and found on on average to be 5 min. Find the probability that a random call made from this booth ① ends less than 5 min ② b/w 5 and 10 min.

0.6321 0.2325

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