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## Normal Distribution

The continuous probability distribution having the probability density function $f(x)$ given by

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}} \quad \text{---} \; (1)$$

where $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$ is known as the normal distribution

Note : 1. The mean & standard deviation of the normal distribution is equal to the mean & standard deviation of the given distribution

### Standard Normal distribution

The standard Normal Variate $z$ is defined as $z = \frac{x-\mu}{\sigma}$

where $\mu$ = mean

$\sigma$ = standard deviation

Also $F(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ is called standard normal probability density function

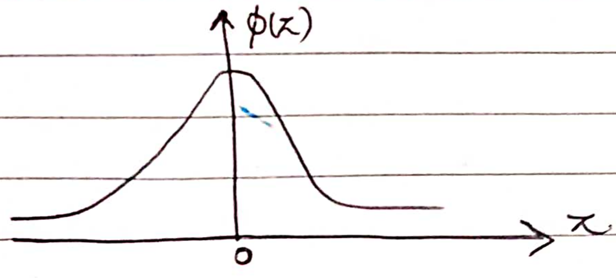
$F(z)$ is same as the Pdf of normal distribution in (1) with $\mu = 0, \sigma = 1$

Thus we can say that the normal ~~probability~~ Pdf with $\mu = 0$ & $\sigma = 1$ is the standard normal Pdf

Also $P(a \le x \le b) = P(z_1 \le z \le z_2) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} dz$ — (2)

$\phi(z)$

The integral in the R.H.S of (2) represents the area bounded by the standard Normal curve $F(z)$ b/w $z = z_1$ & $z = z_2$. $F(z)$ represents a curve which is symmetric about the line $z = 0$. The curve is bell shaped as follows.



In equ (2), In particular, if $z_1 = 0$ & ~~$z_2 = z$~~

Then $\phi(z_2) = P(z \le z_2) = \frac{1}{\sqrt{2\pi}} \int_0^{z_2} e^{-z^2/2} dz$ —(3)

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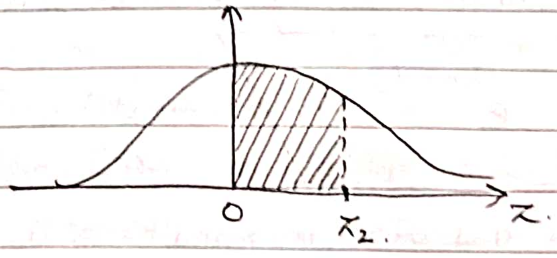
$\phi(z_2)$ ~~dent denotes~~ is area (shaded portion) in the fig.



Clearly 1. $P(-\infty < z < \infty) = 1$ means total area under the bell shaped curve is 1.

2. $P(-\infty < z < 0) = 1/2$

3. $P(0 < z < \infty) = 1/2$

For various values of $z$, the value of integral in equ (3) can be found from the table.

**Note:** Usage of table for various $z$.

1. $P(0 \le z \le z_1) = P(z_1)$
2. $P(-\infty \le z \le z_1) = P(-\infty \le z \le 0) + P(0 \le z \le z_1)$
   $= 0.5 + P(z_1)$
3. $P(z < z_1) = P(-\infty < z < z_1) = 0.5 + P(z_1)$
4. $P(z > z_1) = P(z_1 < z < \infty)$
   $= P(0 < z < \infty) - P(0 < z < z_1)$
   $= 0.5 - P(z_1)$

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Example: 1. $P(Z \geq 0.85) = P(Z \geq 0) - P(Z \leq 0.85)$

$= 0.5 - P(0.85)$

$= 0.5 - 0.3023 = 0.1977$

2. $P(-1.64 < Z < -0.88) = P(1.64 > Z > 0.88)$

$= P(0.88 < Z < 1.64)$

$= P(1.64) - P(0.88)$

$= 0.4495 - 0.3106 = 0.1389$

3. $P(Z \leq -2.43) = P(Z \geq 2.43)$

$= 0.5 - P(2.43)$

$= 0.5 - 0.4925$

$= 0.0075$

4. $P(|Z| \leq 1.94) = P(-1.94 \leq Z \leq 1.94)$

$= 2P(0 \leq Z \leq 1.94)$

$= 2P(1.94)$

$= 2 \times 0.4738 = 0.9476.$

7. If $X$ is a normal variate with mean 30 and SD 5 find (i) $26 \le x \le 40$ $0.7653$
$x \ge 45$ , $|x-30| > 5$ $|x-30| \le 5$
$0.0014$ $0.3175$ $= 0.6826$

Problems DATE:

1. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for
(a) More than 2150 hours $0.0336$ (67) (b) less than 1950 hours $0.0668$ (134)
(c) More than 1920 hours & less than 2160 hours. (0.9544) 1909

HW 2. A sample of 100 dry battery all tested to find the length of life produce the following results, mean = 12 hrs, SD = 3 hrs. Assuming the data is normally distributed what is the percentage of battery all are expected to have a life
(a) more than 15 hrs (b) less than 6 hrs (c) b/w 10 & 14 hrs.
$P(z>1)$ $0.1587$ 15.87% $P(z<-2)$ $0.0228$ 2.28% $-0.6667 < z < 0.6667$ $0.4908$ 49.08%

3. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be ① less than 65 $0.1587$ 159 ② more than 75 $0.1587$ 159 ③ between 65 and 75 $0.6826$ 683

4. In a Normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and SD of the distribution. $\mu = 50, \sigma = 10$

5. In an Examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and SD if the marks are normally distributed. It is given that if $P(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2} \, dx$ then $P(1.2263) = 0.39$ & $P(1.4757) = 0.43$
$X_1$
$\mu = 48.65$
$SD = 9.25$

6. The weight of workers in a large factory are normally distributed with mean 68 kgs and SD 3 kgs. If 80 samples consisting of 35 workers each are chosen, how many of the 80 samples will have the mean b/w 67 and 68.25 kgs
53
Given $P(0 \le z \le 2) = 0.4772$ and $P(0 \le z \le 0.5) = 0.1915$