Model Question Paper-2 with effect from 2019-20 (CBCS Scheme)

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18MAT31

Third Semester B.E.Degree Examination
Transform Calculus, Fourier Series and Numerical Techniques
(Common to all Programmes)

Time: 3 Hrs   Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) Find the Laplace transform of : (i) $3^t + (4t+5)^3$ (ii) $te^{-4t}\sin 3t$ (ii) $(\cos at - \sin bt)/t$ **(10 Marks)**

(b) The triangular wave function $f(t)$ with period "2*a*" is defined by $f(t) = \begin{cases} t, & 0 \le t < a \\ 2a - t, & a \le t < 2a \end{cases}$

Show that $L\{f(t)\} = (1/s^2)\tanh(as/2)$. **(05 Marks)**

(c) Using Laplace transform method, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t$, $y(0) = 0 = y'(0)$. **(05 Marks)**

OR

2. (a) Find the inverse Laplace transform of (i) $\left\{\frac{1}{s(s+1)}\right\}$ (ii) $\{(s+1)/(s^2+6s+9)\}$

(iii) $[\log\{(s+a)/(s+b)\}]$ **(10 Marks)**

(b) Express $f(t) = \begin{cases} \sin t, & 0 < t \le \pi/2 \\ \cos t & t > \pi/2 \end{cases}$ in terms Heaviside's unit step function and hence find its Laplace transform. **(05 Marks)**

(c) Find the Laplace transform of $\frac{4}{(s^2+2s+5)^2}$, using convolution theorem. **(05 Marks)**

Module-2

3. (a) An alternating current $I(x)$ after passing through a rectifier has the form $I(x) = \begin{cases} I_0 \sin x, \ for & 0 \le x < \pi \\ 0, \quad for & \pi < x \le 2\pi, \end{cases}$

where I_0 is the maximum current and the period is 2π. Express $I(x)$ as a Fourier series. **(07 Marks)**

(b) Find the half-range sine series of $f(x) = \frac{\sinh ax}{\sinh a\pi}$ the interval $(0, \pi)$. **(06 Marks)**

(c) Find the Fourier series expansion of $f(x) = x(1-x)(2-x)$ the interval $0 \le x \le 2$. Hence deduce the sum of the series that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots$ **(07 Marks)**

OR

4. (a) In an electrical research laboratory, scientists have designed a generator which can generate the following currents at different time instant t, in the period T:

Time t (in sec)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
$f(x)$	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Determine the direct current part and amplitude of the first harmonic from the above data. **(07 Marks)**

(b) Find the half-range sine series of $f(x)=\begin{cases}\sin x & for \quad 0\le x<\pi/4\\ \cos x, & for \quad \pi/4<x\le\pi/2\end{cases}$ **(06 Marks)**

(c) Obtain the Fourier series of $f(x)=x(2\pi-x)$ valid in the interval $(0,2\pi)$. **(07 Marks)**

Module-3

5. (a) If $f(x)=\begin{cases}a^2-x^2, & for\ |x|\le a\\ 0 & for\ |x|>a\end{cases}$, find the infinite Fourier transform of *f(x)* and hence evaluate

$\int_0^\infty \frac{\sin x - x\cos x}{x^3}dx$ **(07 Marks)**

(b) Find the Fourier sine transform of $f(x)=\begin{cases}x, & if\ 0<x<1\\ 2-x, & if\ 1<x<2\\ 0, & if\ x>2\end{cases}$ **(06 Marks)**

(c) Solve $u_{n+2}+6u_{n+1}+9u_n=2^n, u_0=0=u_1$, by using z-transforms. **(07 Marks)**

OR

6. (a) If $f(x)=\begin{cases}1, & for\ |x|\le a\\ 0 & for\ |x|>a\end{cases}$, find the infinite Fourier transform of *f(x)* and hence evaluate

$\int_0^\infty \frac{\sin x}{x}dx$ **(07 Marks)**

(b) Find the z-transform of $2n+\sin(n\pi/4)+1$ **(06 Marks)**

(c) Find the inverse z-transform of $18z^2/[(2z-1)(4z+1)]$ **(07 Marks)**

Module-4

7. (a) Solve $\frac{dy}{dx}=x^3+y, y(1)=1$ using Taylor's series method considering up to fourth degree terms and, find the $y(1.1)$. **(07 Marks)**

(b) Use Runge - Kutta method of fourth order to solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$, to find $y(0.2)$. (Take $h = 0.2$). **(06 Marks)**

(c) Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1, y(1.1) = 1.2330, y(1.2) = 1.5480, \& \; y(1.3) = 1.9790$ find $y(1.4)$, using Adam-Bashforth predictor-corrector method. **(07 Marks)**

OR

8\. (a) Use modified Euler's method to compute $y(0.2)$, given $\frac{dy}{dx} - xy^2 = 0$ under the initial condition $y(0) = 2$. Perform three iterations at each step, taking $h = 0.1$. **(07 Marks)**

(b) Use fourth order Runge - Kutta method, to find $y(0.2)$ with $h = 0.2$, given $\frac{dy}{dx} = \sqrt{x+y}, y(0) = 1$, **(06Marks)**

(c) Apply Milne's predictor-corrector formulae to compute $y(2.0)$ given $\frac{dy}{dx} = \frac{1}{2}(x+y)$ with **(07 Marks)**

x	0.0	0.5	1.0	1.5
y	2.0000	2.6360	3.5950	4.9680

Module-5

9\. (a) Using Runge - Kutta method, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$, for $x = 0.2$, correct to four decimal places, using initial conditions $y(0) = 1, y'(0) = 0$. **(07 Marks)**

(b) Derive Euler's equation in the standard form viz., $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$ **(06 Marks)**

(c) Find the extremal of the functional $\int_0^{\pi}\left(y'^2 - y^2 + 4y\cos x\right)dx$; $y(0) = 0 = y(\pi)$ **(07 Marks)**

OR

10\. (a) Given the differential equation $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x = 0$ and the following table of initial values:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	2.0657

compute $y(1.4)$ by applying Milne's predictor-corrector method. **(07 Marks)**

(b) On what curves can the functional $\int_0^{\pi/2}\left(y'^2 - y^2 + 2xy\right)dx$, ; $y(0) = 0, y(\pi/2) = 0$ be extremized? **(06 Marks)**

(c) Prove that geodesics of a plane surface are straight lines. **(07 Marks)**

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