# Model Question Paper-1 with effect from 2019-20 (CBCS Scheme)

USN | | | | | | | | | | |

**18MAT41**

## Fourth Semester B.E. Degree Examination
## Complex Analysis, Probability and Statistical Methods
(Common to all Programmes)

Time: 3 Hrs Max.Marks: 100

**Note: Answer any FIVE full questions, choosing at least ONE question from each module.**
**Use of statistical tables allowed.**

### Module-1

1 (a) Show that $w = f(z) = z + e^z$ is analytic and hence find $dw/dz$. **(06 Marks)**

(b) Derive Cauchy-Riemann equation in cartesian form. **(07Marks)**

(c) Find the analytic function $f(z) = u + iv$, given $v = [r - (1/r)]\sin\theta, r \neq 0$. **(07Marks)**

**OR**

2 (a) If $f(z) = u(x,y) + iv(x,y)$ is an analytic function, show that the family of curves $u(x,y) = c_1$ and $v(x,y) = c_2$, $c_1$ & $c_2$ being constants, intersect each other orthogonally. **(06 Marks)**

(b) If $f(z)$ is analytic, show that $[(\partial^2/\partial x^2) + (\partial^2/\partial y^2)]\ |f(z)|^2 = 4|f'(z)|^2$. **(07Marks)**

(c) Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic. Also, determine the corresponding analytic function. **(07Marks)**

### Module-2

3 (a) State and prove Cauchy's theorem. **(06 Marks)**

(b) Find the image in the $w$- plane bounded by the lines $x = 1, y = 1, x + y = 1$under the transformation $w = z^2$. **(07Marks)**

(c) Find the bilinear transformation which maps the points $z = 0,1,\infty$ into the points $w = -5,-1,3$, respectively. What are the invariant points under this transformation? **(07Marks)**

**OR**

4 (a) Evaluate: $\int_0^{2+i} (\bar{z})^2\, dz$ along the line $x = 2y$ . **(06 Marks)**

(b) Evaluate: $\int_C \frac{e^z}{(z+1)(z-2)}\, dz$ where $C$ is the circle $|z| = 3$. **(07Marks)**

(c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0,1,\infty$ respectively. What are the invariant points under this transformation? **(07Marks)**

## Module-3

5 (a) A random variable $X$ has the following probability function for various values of $x$: **(06 Marks)**

| $X(=x_i)$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(x)$ | 0.1 | $k$ | 0.2 | $2k$ | 0.3 | $k$ |

Find (i) the value of $k$ (ii) $P(x<1)$ (iii)$P(x\geq -1)$.

(b) The probability of germination of a seed in a packet of seeds is found to be 0.7. If 10 seeds are taken for experimenting on germination in a laboratory, find the probability that (i) 8 seeds germinate (ii) at least 8 seeds germinate (iii) at most 8 seeds germinate. **(07Marks)**

(c) If the life time of a certain types electric bulbs of a particular brand was distributed normally with an average life of 2000 hours and S.D. 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2500 hours (ii) less than 1950 hours (iii) between 1900 and 2100 hours. **(07Marks)**

**OR**

6 (a) The probability density function of a random variable $X(=x)$ is $f(x)=\begin{cases}kx^2, & 0<x<3\\ 0, & otherwise\end{cases}$. **(06 Marks)**

Find (i) the value of $k$ (ii) $P(1<x<2)$ and (iii)$P(x\leq 1)$.

(b) In a certain factory manufacturing the razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i) no defective ii) one defective iii) two defective blades, in a consignment of 10,000 packet. **(07Marks)**

(c) The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 minutes (ii) between 5 and 10 minutes. **(07Marks)**

## Module-4

7 (a) The following are the marks of 8 students in Statistics and Mathematics. **(06 Marks)**

| *Marks in Statistics* | 25 | 43 | 27 | 35 | 54 | 61 | 37 | 45 |
|---|---|---|---|---|---|---|---|---|
| *Marks in Mathematics* | 35 | 47 | 20 | 37 | 63 | 54 | 28 | 40 |

Calculate the rank correlation coefficient.

(b) Fit a best fitting curve in the form $y = ax^b$ for the following data: **(07Marks)**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 2.98 | 4.26 | 5.21 | 6.1 | 6.8 | 7.5 |

Calculate the value of $y$ when $x = 3.5$.

(c) With usual notation, compute $\bar{x}, \bar{y}$ and $r$ from the following lines of regression: **(07Marks)**

$y = 0.516x + 33.73$ and $x = 0.512\ y$ +32.52.

**OR**

8 (a) In a bivariate distribution, it is found that $\sigma_x = \sigma_y$ and the acute angle between the lines of regression is $tan^{-1}(3)$. Find the correlation coefficient. **(06 Marks)**

(b) Find the coefficient of correlation between the industrial production and export, using the following table; **(07Marks)**

| *Production (in Lakh tons)* | 55 | 56 | 58 | 59 | 60 | 60 | 60 |
|---|---|---|---|---|---|---|---|
| *Exports (in Lakh tons)* | 35 | 38 | 38 | 39 | 44 | 43 | 45 |

(c) Following are the measurements of air velocity ($x$) and evaporation coefficient ($y$) of burning fuel droplets in an impulse engine: **(07Marks)**

| $x$ | 20 | 60 | 100 | 140 | 180 | 220 |
|---|---|---|---|---|---|---|
| $y$ | 0.18 | 0.37 | 0.35 | 0.78 | 0.56 | 0.75 |

Find a best fitting parabola $y = ax^2 + bx + c$ to the above data and hence estimate $y$ when $x = 99$.

## Module-5

9 (a) Determine (i) marginal distribution (ii) covariance between the discrete random variables $X$ and $Y$, of the jont probsbility distribution: **(06 Marks)**

| X \ Y | 3 | 4 | 5 |
|---|---|---|---|
| 2 | 1/6 | 1/6 | 1/6 |
| 5 | 1/12 | 1/12 | 1/12 |
| 7 | 1/12 | 1/12 | 1/12 |

(b) The mean life time of a sample of 100 fluorescent tube lights manufactured by a company is found to be 1570 hrs with a standard deviation of 120 hrs. Test the hypothesis that the mean life-time of the lights produced by the company is 1600 hrs at 0.01 level of significance. **(07Marks)**

(c) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure $5, 3, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4$. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 d.f. is 2.201). **(07Marks)**

**OR**

10 (a) Explain the terms: (i) Null hypothesis (ii) Confidence intervals (iii)Type I and Type II errors. **(06 Marks)**

(b) It is claimed that a random sample of 49 tyres has a mean life of 15,200 kms. Is the sample drawn from a population whose mean is 15,150 kms and whose standard deviation is 1,200 kms? Test the significance at 0.05 level. **(07Marks)**

(c) Fit a binomial distribution for the data **(07Marks)**

| *No. of Heads* | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| *Frequency* | 122 | 60 | 15 | 2 | 1 |

and also test the goodness of fit given that $\chi^2_{0.05}$= 7.815 for 3 d.f.