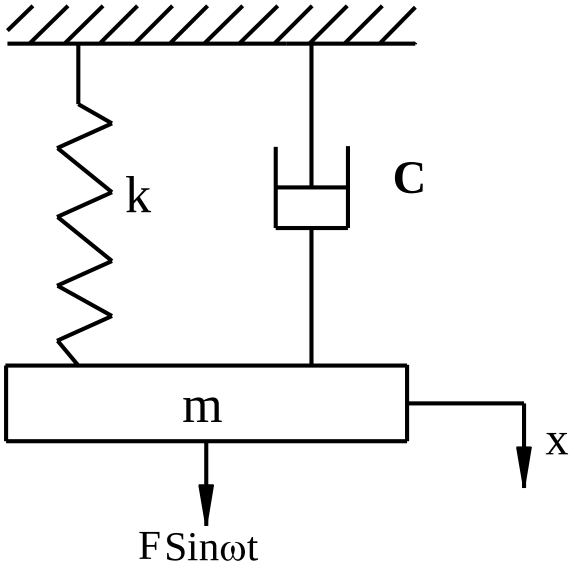
Mechanical Vibrations (16 ME 72)
Chapter 3: Forced vibrations

- When a periodic external force acts on the vibrating system, it is called ***forced vibrations***.
- The frequency of vibration of such a system is equal to the frequency of the external excitation force.

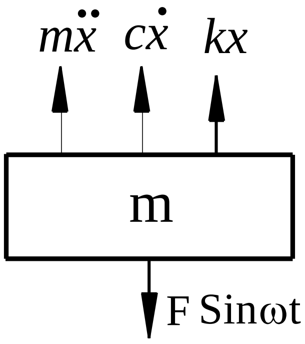
Ex: *Rotating or reciprocating unbalance in machine tools, Ringing of electric bell, etc.*

- If the frequency of the impressed force becomes equal to the natural frequency of the system, it is said to be under ***resonance***.

Forced vibration of single degree of freedom system



Forced Vibration



Free Body diagrams

Consider a spring-mass-damper system having viscous damping excited by a sinusoidal force $F\sin\omega t$ where;

F_o = Amplitude of force &

ω = Circular frequency of the external force.

Forced vibrations with damping

When the mass is displaced from its equilibrium position by a distance 'x'

& released, after a time $'t'$, for equilibrium,

Inertia force + Damper force + Spring force = External force

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = F\sin\omega t$$

This is a second order linear differential equation with constant coefficients.

The general solution of the above equation is of the form $x = x_c + x_p$ where;

$\rightarrow x_c$ is the complementary solution of the equation $m\ddot{x} + c\dot{x} + kx = 0$

which is transient & vanishes with time. ***Hence it can be ignored.***

$\rightarrow x_p$ is the particular solution of the form $x_p = A\sin(\omega t - \phi)$

where $'A'$ ***is the amplitude of steady state vibration*** & $'\phi'$ ***is the phase of displacement w.r.t harmonic force.***

Forced vibrations with damping

The equilibrium equation is $\boldsymbol{m\ddot{x} + c\dot{x} + kx = Fsin\omega t \cdots (i)}$

Neglecting the transient response, $x = A\sin(\omega t - \phi)$

$\Rightarrow Velocity\ \dot{x} = \omega A\cos(\omega t - \phi) = \omega A\sin\left(\omega t - \phi + \frac{\pi}{2}\right)$ and

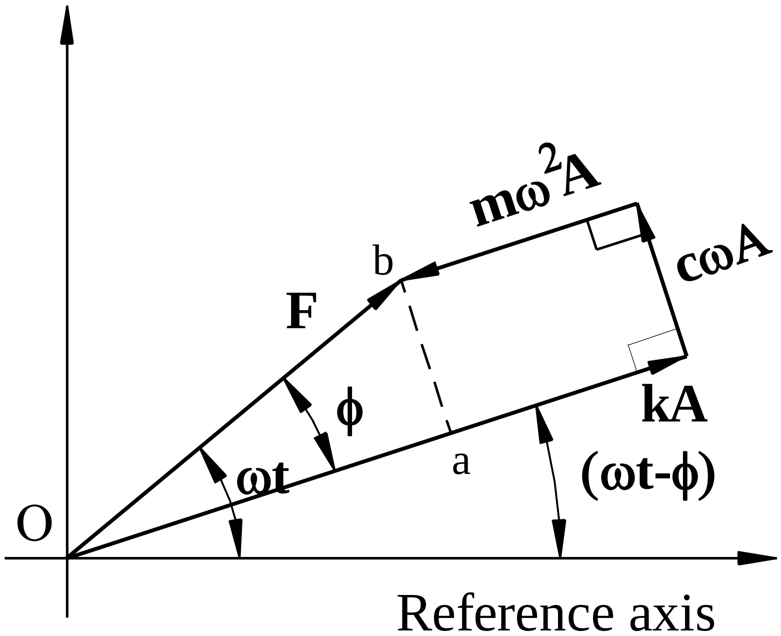
Acceleration $\ddot{x} = -\omega^2 A\sin(\omega t - \phi) = \omega^2 A\sin\left(\omega t - \phi + \pi\right)$

Substituting in (i), the equilibrium equation becomes,

$$m\omega^2 A\sin\left(\omega t - \phi + \pi\right) + c\omega A\sin\left(\omega t - \phi + \frac{\pi}{2}\right) + kA\sin(\omega t - \phi) = F\sin\omega t$$

The above forces may be represented by vectors drawn in order such that they form a closed polygon. Damping force is perpendicular to spring force & the inertia force is perpendicular to damping force.

Vector representation of forces



$$A\sin\left(\omega t-\phi+\frac{\pi}{2}\right) \text{ and}$$

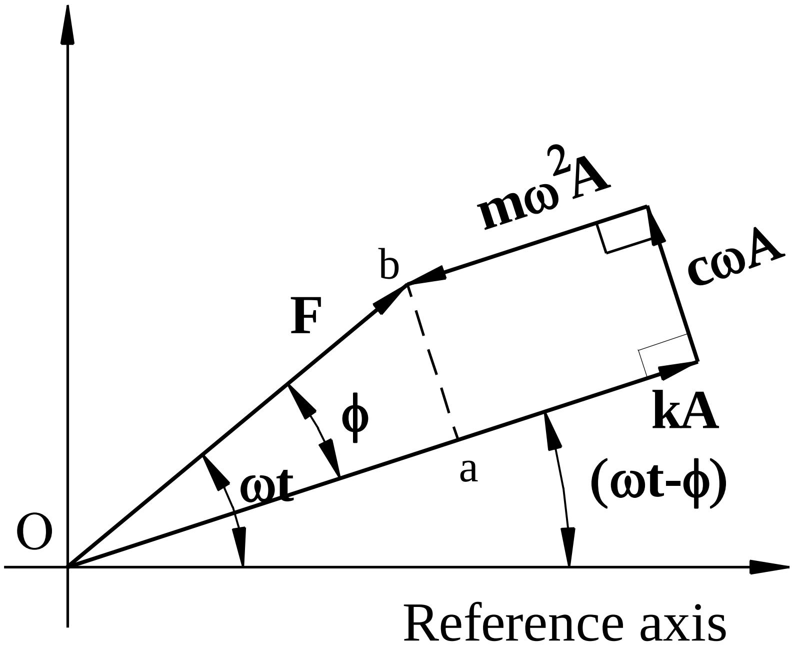
$$=\omega^2 A\sin(\omega t-\phi+\pi)$$

equation becomes,

$$m\omega^2 A\sin(\omega t-\phi+\pi)+c\omega A\sin\left(\omega t-\phi+\frac{\pi}{2}\right)+kA\sin(\omega t-\phi)=F_o\sin\omega t$$

The above forces may be represented by vectors drawn in order such that they form a closed polygon. Damping force is perpendicular to spring force & the inertia force is perpendicular to damping force.

Vector representation of forces



From the triangle ***oab*** in the fig,

$$F^2 = (kA - m\omega^2 A)^2 + (c\omega A)^2$$

$$= A^2\left[(k - m\omega^2)^2 + (c\omega)^2\right]$$

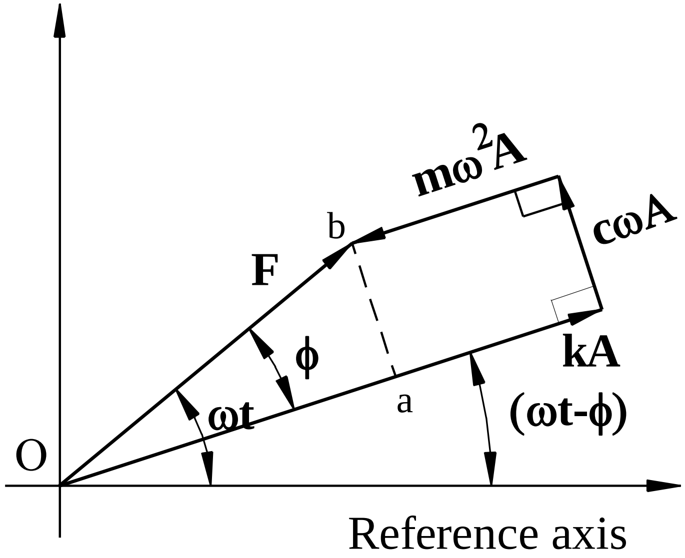
$$A = \frac{F}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$$

Taking k outside the sq root,

$$A = \frac{F/k}{\sqrt{\left(1 - m\omega^2/k\right)^2 + \left(c\omega/k\right)^2}}$$

Also $k = m\omega_n^2, c = 2\zeta m\omega_n$

$$\therefore A = \frac{F/k}{\sqrt{\left(1 - \left(\omega/\omega_n\right)^2\right)^2 + \left(2\zeta\omega/\omega_n\right)^2}}$$



Steady state amplitude & phase angle of forced vibrations :

$$A = \frac{\left(F/k\right)}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

where $\left(\omega/\omega_n\right) = r$, the frequency ratio

From the fig, phase angle between force & displacement is $\phi = \tan^{-1}\left(\dfrac{2\zeta r}{1-r^2}\right)$

(i) At resonance, $\omega = \omega_n \Rightarrow r = 1 \ \therefore A = \dfrac{\left(F/k\right)}{2\zeta}, \ \phi = \tan^{-1}\infty = 90^0$

(ii) When $\zeta = 0$, ***(undamped),*** $A = \dfrac{\left(F/k\right)}{\pm(1-r^2)}, \ \phi = \tan^{-1} 0 = 0^0$

Magnification (Amplification) factor MF

$$We\ know\ that\ amplitude\ A = \frac{F/k}{\sqrt{\left(1-\left(\omega/\omega_n\right)^2\right)^2 + \left(2\zeta\omega/\omega_n\right)^2}}$$

Put $\left(\omega/\omega_n\right) = r,$ the frequency ratio &

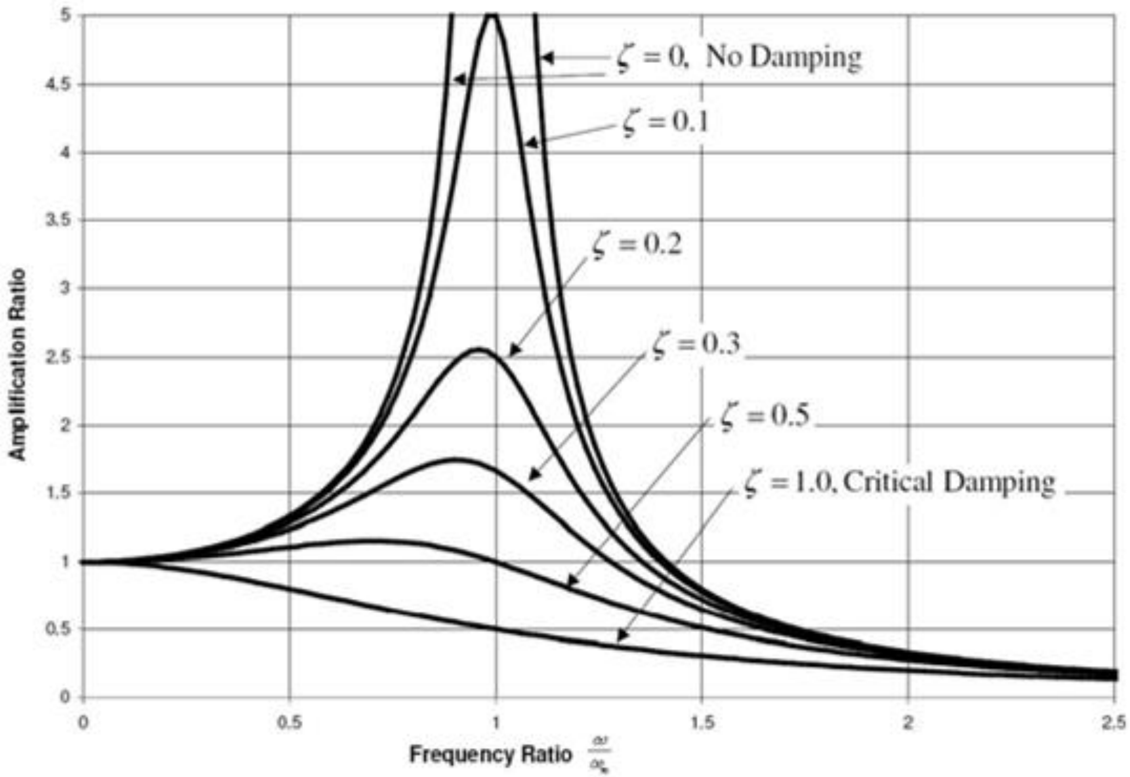
$\left(F/k\right) = A_{st},$ the static deflection under load F,

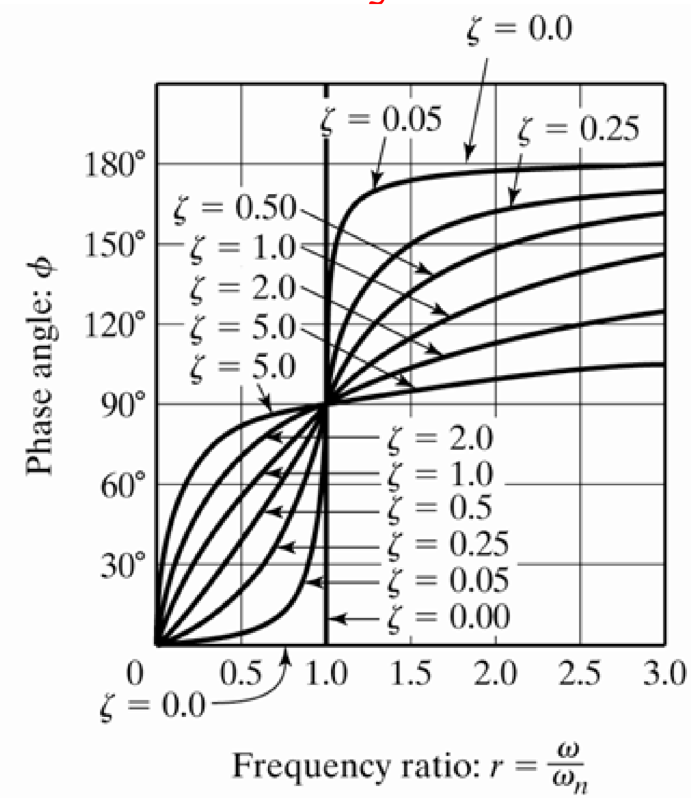
$$\therefore A = \frac{A_{st}}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \Rightarrow \frac{\boldsymbol{A}}{\boldsymbol{A_{st}}} = \frac{\boldsymbol{1}}{\sqrt{\left(\boldsymbol{1-r^2}\right)^{\boldsymbol{2}} + \left(\boldsymbol{2\zeta r}\right)^{\boldsymbol{2}}}}$$

The dimensionless ratio $\left(\dfrac{A}{A_{st}}\right)$ is called the ***Magnification factor (MF)***

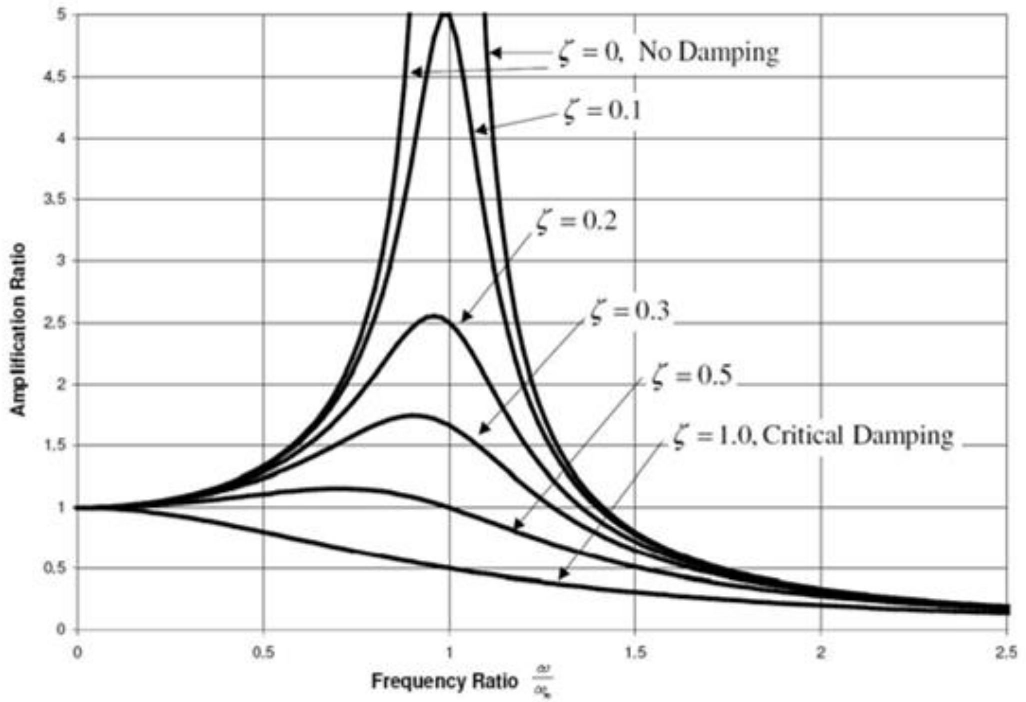
& it is a function of frequency ratio & damping factor.

Plot of MF vs. r for different values of ζ





Important observations of MF vs. Frequency ratio



- Any amount of damping ($\zeta > 0$) reduces the magnification factor (MF) for all values of forcing frequency.
- The amplitude of vibration is infinite at resonant frequency & zero damping factor.
- The peak amplitude for any amount of damping occurs slightly before *r=1*
- For $\zeta > 1/\sqrt{2}$, the graphs of MF decreases with increasing values of r.

Frequency corresponding to peak amplitude

It is observed that the peak amplitude for any amount of damping occurs slightly before $r = 1$ or resonant frequency.

$$A = \frac{A_{st}}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}.$$ Max value of A occurs when the denominator

$\left[\left(1-r^2\right)^2 + \left(2\zeta r\right)^2\right]$ is minimum. $\Rightarrow \dfrac{d}{dr}\left[\left(1-r^2\right)^2 + \left(2\zeta r\right)^2\right] = 0$

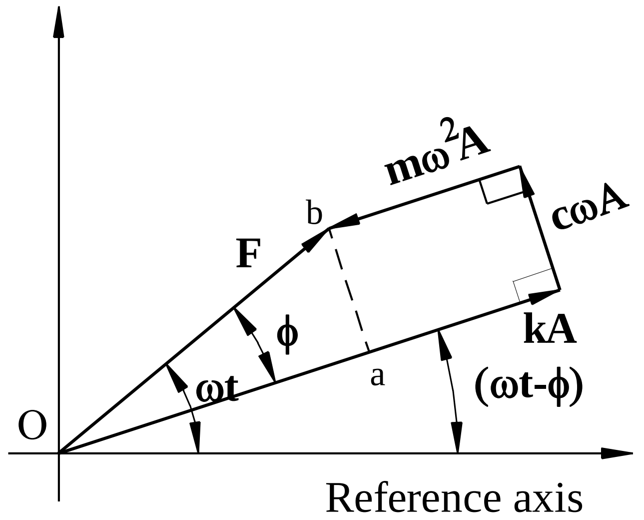
$$i.e. \left[2(1-r^2)\times 2r + 4\zeta^2 \times 2r\right] = 0$$

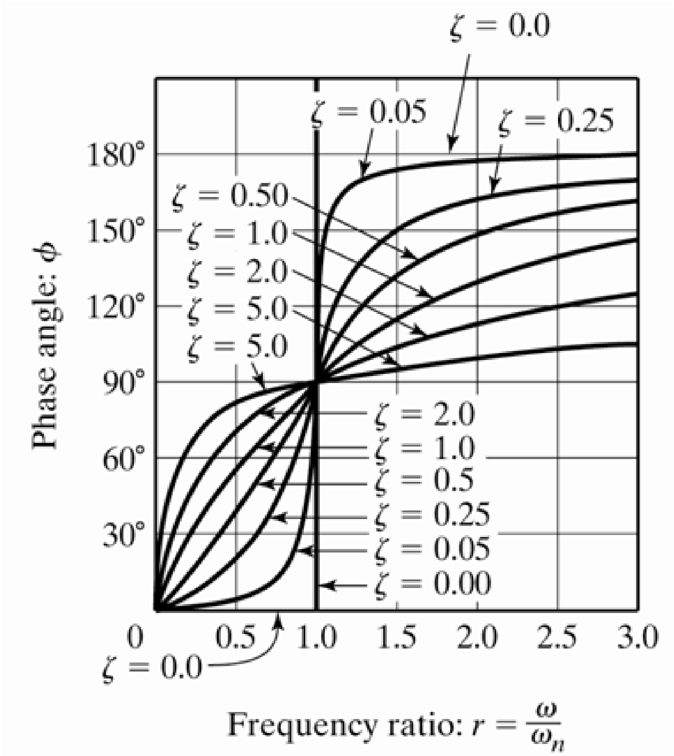
$\Rightarrow (1-r^2) + 2\zeta^2 = 0$ *Or* $r = \sqrt{(1-2\zeta^2)}$ But $r = \dfrac{\omega}{\omega_n}$

$\therefore$ Frequency of external excitation corresponding to peak amplitude

$$\boldsymbol{\omega_p = \sqrt{(1-2\zeta^2)}}$$

Important observations of ϕ vs. Frequency ratio





1) For $\zeta > 0$ and $0 < r < 1$ the phase angle is given by $0^0 < \phi < 90^0$, implying that the response lags excitation.
2) For $\zeta > 0$ and $r > 1$, the phase angle is given by $90^0 < \phi < 180^0$, implying that the response leads excitation.
3) For $\zeta > 0$ and $r = 1$, the phase angle is $\phi = 90^0$ implying that the phase difference between the excitation and response is 90^0.
4) For $\zeta > 0$ and large values of r, the phase angle ϕ approaches 180^0 implying that the response and excitation are out of phase.

Numerical problem 1

A machine part of mass 2.5 Kg vibrates in a viscous medium. A harmonic exciting force of 30 N acts on the part and causes resonant amplitude of 14 mm with a period of 0.22sec. Determine the damping coefficient.

If the frequency of the exciting force is changed to 4Hz, determine the increase in the amplitude of forced vibration upon removal of the damper.

Data : $m = 2.5\ kg,\ F = 30\ N,\ A_{res} = 14\ mm,\ T = 0.22\ \sec$

Solution : $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.22} = \boldsymbol{28.56\ rad / sec}$

(i) At resonance : $\omega = \omega_n \Rightarrow r = \left(\dfrac{\omega}{\omega_n}\right) = 1$

$\therefore$ Stiffness of spring $k = m\omega_n^2 = 2.5 \times 28.56^2 = \boldsymbol{2039\ N / m}$

$$Amplitude\ A = \frac{F/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad If\ \ r = 1,\ A = A_{res} = \frac{F/k}{2\zeta}$$

$$Given\ A_{res} = 14\ mm = 0.014\ m,\ 0.014 = \frac{\left(30/2039\right)}{2\zeta} \Rightarrow \zeta = \boldsymbol{0.526}$$

Damping coefficient c = $2\zeta m\omega_n$

$i.e.\ \ \boldsymbol{c} = 2 \times 0.526 \times 2.5 \times 28.56 = \boldsymbol{75.11\ N\text{ - }sec / m}$

(ii) When f = 4 Hz : $\omega = 2\pi f = 2\pi \times 4 = \boldsymbol{25.13\ rad / sec}$

$$\omega_n = 28.56\ rad/\sec \Rightarrow \ frequency\ ratio\ r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{25.13}{28.56}\right) = 0.88$$

Amplitude of vibration with damper is ;

$$A = \frac{F/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\left(30/2039\right)}{\sqrt{(1-0.88^2)^2 + (2\times 0.526 \times 0.88)^2}} = \boldsymbol{0.0154\ m}$$

Amplitude of vibration when damper is removed ($\zeta = 0$) is;

$$A' = \frac{F/k}{(1-r^2)} = \frac{\left(30/2039\right)}{(1-0.88^2)} = \boldsymbol{0.0652\ m}$$

$\therefore$ ***Increase in amplitude upon removal of damper***

$$(A' - A) = (0.0652 - 0.0154) = \boldsymbol{0.05\ m = 50\ mm}$$

Numerical Problem 2

A body having a mass of 15 kg is suspended from a spring which deflects 12mm due to the weight of the mass. Determine the frequency of free vibrations. What viscous damping force is needed to make the motion ***aperiodic*** at a speed of 1 mm/sec.

If when, damped to this extent, a disturbing force having a maximum value of 100 N and vibrating at 6 Hz is made to act on the body. Determine the amplitude of ultimate motion.

Data : $m = 15\ kg,\ \Delta = 12\ mm = 0.012m,\ \dot{x} = 1\ mm/\sec,$

$F = 100\ N,\ f = 6\ Hz$

Solution :

(i) Frequency of free vibrations :

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{0.012}} = \boldsymbol{28.6\ rad\ /\ sec}$$

To make the motion ***aperiodic***, $\zeta \geq 1$ *i.e.* at least, $\zeta = 1$ or $c = c_c$

$\Rightarrow$ **Damping coefficient** $c_c = 2m\omega_n = 2 \times 15 \times 28.6 = \boldsymbol{858N\ /\ m\ /\ sec}$

$= \boldsymbol{0.858N\ /\ mm\ /\ sec}$ *i.e. a force of* $0.858\ N$ *is required at a rate of* $1\ mm/\sec$ *to make the motion aperiodic.*

(ii) Amplitude of forced vibrations :

$$\omega = 2\pi f = 2\pi \times 6 = \boldsymbol{37.7\ rad\ /\ sec}$$

$$\text{Ratio of frequencies } r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{37.7}{28.6}\right) = \boldsymbol{1.318}$$

$$Stiffness\ of\ the\ spring\ k = m\omega_n^2 = 15 \times 28.6^2 = \boldsymbol{12269.4N\ /\ m}$$

$$Damping\ ratio\ \zeta = 1$$

(iii) Amplitude of vibration with damper :

$$A = \frac{F/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\left(100/12269.4\right)}{\sqrt{(1-1.318^2)^2 + (2\times 1\times 1.318)^2}} = \boldsymbol{0.022\ m}$$

Numerical Problem 3

A mass of 10 kg is suspended from one end of a helical spring, the other end is fixed. The stiffness of spring is 10 N/mm. The viscous damping causes the amplitude to decrease to $1/10^{th}$ of initial value in four complete oscillations. If a periodic force of 150 cos 50t N is applied at the mass, determine the amplitude and phase angle of the resulting motion. What is its value at resonance?.

Data : $m = 10\ kg,\ F = 150\ N,\ \omega = 50\ rad/\sec,$

$k = 10 N/mm = 10000\ N/m,\ x_5 = 0.1 x_1, n = 4$

Solution :

$$\text{Natural frequency } \omega_n = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{10000}{10}\right)} = \boldsymbol{31.62\ rad\ /\ sec}$$

$$\text{Log decrement } \delta = \frac{1}{n}\ln\left(\frac{x_1}{x_{n+1}}\right) = \frac{1}{4}\ln\left(\frac{x_1}{0.1x_1}\right) = \frac{1}{4}\ln(10) = \boldsymbol{0.5756}$$

$$\text{Damping factor } \zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = \frac{0.5756}{\sqrt{4\pi^2 + 0.5756^2}} = \boldsymbol{0.0912}$$

$$\text{Frequency ratio } r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{50}{31.62}\right) = \boldsymbol{1.58}$$

(i) Amplitude of motion : $Amplitude\ A = \dfrac{F/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$$A = \frac{150/10000}{\sqrt{(1-1.58^2)^2 + (2\times 0.0912\times 1.58)^2}} = 9.82\times 10^{-3} m = \boldsymbol{9.82\ mm}$$

(ii) Phase angle between force & displacement :

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right) = \tan^{-1}\left(\frac{2\times 0.0912\times 1.58}{1 - 1.58^2}\right) = -10.9^0$$

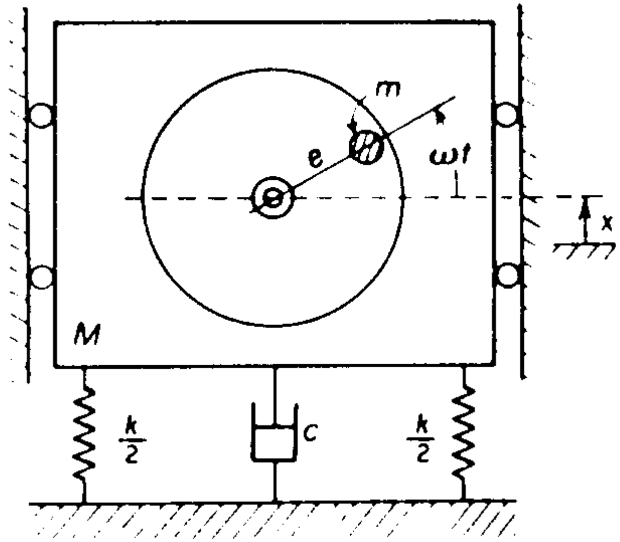
-ve sign indicates that the force lags the displacement.

(iii) Amplitude at resonance : $\omega = \omega_n \Rightarrow r = 1$

$$A = \frac{\left(F/k\right)}{2\zeta} = \frac{\left(150/10000\right)}{2\times 0.0912} = \boldsymbol{0.0822\ m = 82.2\ mm}$$

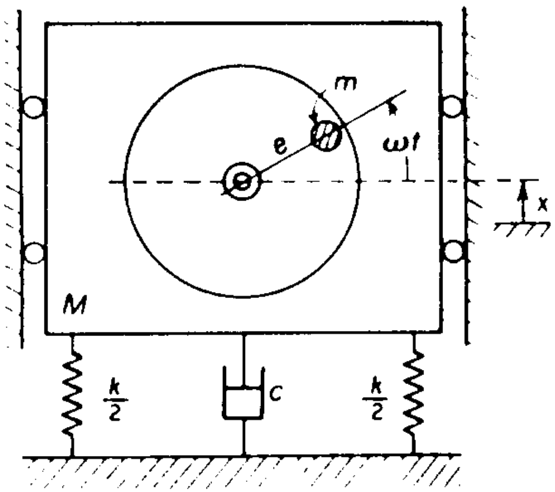
Rotating Unbalance

- Unbalance in rotating machines is a common source of vibration excitation.
- We consider here a spring (k) & mass (M) constrained to move in the vertical direction and excited by a rotating machine that is unbalanced, as shown in Fig.

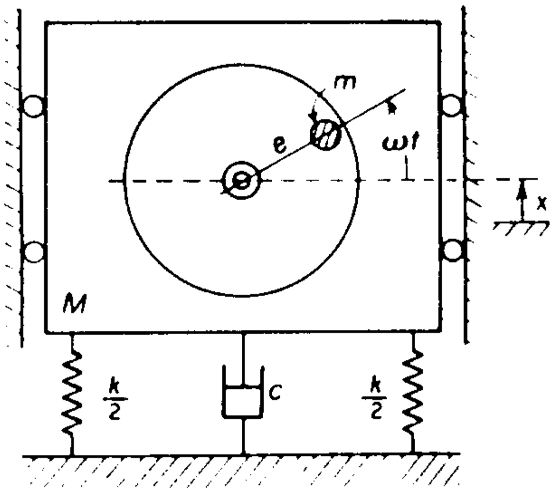


Rotating Unbalance

- The unbalance is represented by an eccentric mass ***m*** with eccentricity ***e*** that is rotating with angular velocity **ω** .
- By letting ***x*** be the displacement of the non rotating mass *(M - m) from* the static equilibrium position, the displacement of *m* is $(x+e\sin\omega t)$



Rotating Unbalance



The equation of motion is then:

$$(M-m)\frac{d^2x}{dt^2}+m\frac{d^2}{dt^2}(x+e\sin\omega t)+c\frac{dx}{dt}+kx=0$$

$$(M-m)\ddot{x}+m\ddot{x}-m\omega^2 e\sin\omega t+c\dot{x}+kx=0$$

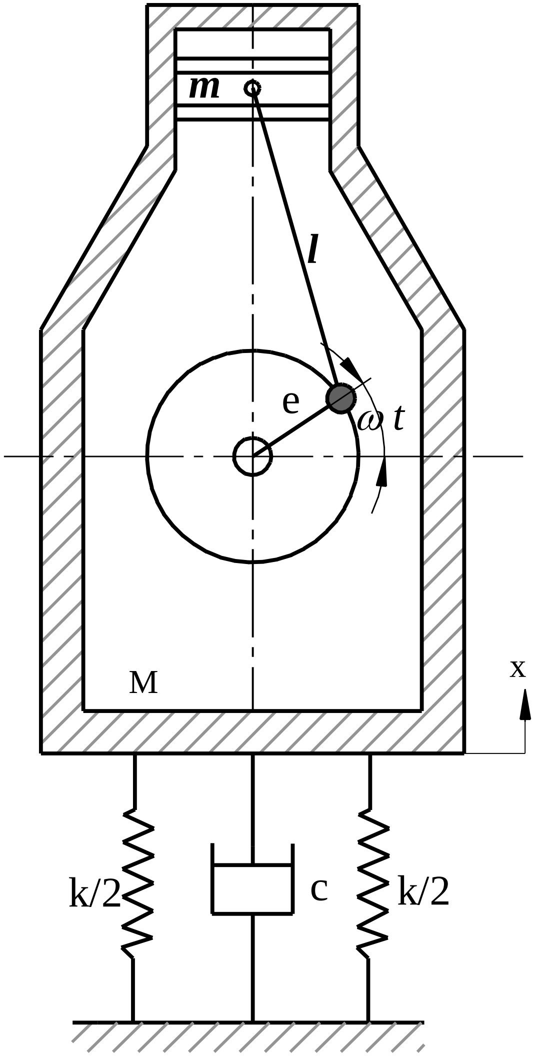
$$\Rightarrow \boldsymbol{M\ddot{x}+c\dot{x}+kx=(m\omega^2 e)\sin\omega t.}$$

Comparing with the eqn of forced vibration $M\ddot{x}+c\dot{x}+kx=F\sin\omega t,$

the force due to rotating unbalance is $\boldsymbol{F=m\omega^2 e}$

Note : m *is in kg,* $\boldsymbol{\omega}$ *in rad* / sec & ***e*** *in meters*

Reciprocating Unbalance



In this case, the harmonic unbalance force is;

$$= \; m\omega^2 e\left[\sin\omega t + \frac{e}{l}\sin 2\omega t\right]$$

$$where\ m = \ mass\ of\ \ piston,$$

$$e = crank\ \ radius = \frac{stroke\ \ length}{2}$$

$$l = length\ of\ \ connecting\ \ rod\,,$$

$$\omega = angular\ \ velocity\ \ of\ \ crank.$$

$As\ e << l,\ \left(\dfrac{e}{l}\right)$ may be neglected,

$harmonic\ \ force\ \ is\ \ (m\omega^2 e)\sin\omega t \Rightarrow$ $\boldsymbol{F = (m\omega^2 e)}$

$which\ \ is\ \ same\ \ as\ \ rotating\ \ unbalance.$

$\boldsymbol{Note : m}\ \ is\ in\ kg\,,\ \boldsymbol{\omega}\ \ in\ rad\ /\ \sec\ \&\ \boldsymbol{e}\ in\ meters$

Numerical Problem 1
(Reciprocating unbalance)

A single cylinder vertical diesel engine has a mass of 400 kg and is mounted on a steel chassis frame. The static deflection owing to weight of the chassis is 2.4 mm. The reciprocating masses of the engine amount to 18 kg and the stroke of the engine is 160 mm. A dashpot with a damping coefficient of 2 N/mm is also used to dampen the vibrations. In the steady-state of vibrations, determine;

(i) Amplitude of vibration at 500 rpm of driving shaft

(ii) The speed of the driving shaft at resonance

Data : $M = 400\ kg,\ m = 18\ kg,\ N = 500\ rpm, \Delta = 2.4 = 2.4\times10^{-3} m$

Stroke of piston =160 mm $\Rightarrow$ crank radius $e = 80\ mm = 0.08\ m$

$c = 2N/mm/\sec = 2000N/m/\sec$

Solution :

$$Natural\ frequency\ \omega_n = \sqrt{\left(\frac{g}{\Delta}\right)} = \sqrt{\left(\frac{9.81}{2.4\times10^{-3}}\right)} = \mathbf{63.93\ rad/sec}$$

$$Stiffness\ of\ chassis\ k = M\omega_n^2 = 400\times63.93^2 = \mathbf{1.635\times10^6\ N/m}$$

$$Forcing\ frequency\ \omega = \frac{2\pi N}{60} = \frac{2\pi\times500}{60} = \mathbf{52.36\ rad/sec}$$

$$Frequency\ ratio\ r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{52.36}{63.93}\right) = \mathbf{0.82}$$

$$Damping\ factor\ \zeta = \frac{c}{c_c} = \frac{c}{2M\omega_n} = \frac{2000}{2\times400\times63.93} = \mathbf{0.0391}$$

$$Unbalanced\ force\ F = m\omega^2 e = 18\times(52.36)^2\times0.08 = \mathbf{3948\ N}$$

(i) Amplitude of motion at 500 rpm :

$$Amplitude\ A = \frac{F/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$A = \frac{3948/1.635\times10^6}{\sqrt{(1-0.82^2)^2 + (2\times0.0391\times0.82)^2}} = 8.1\times10^{-4} m = \boldsymbol{0.81\ mm}$$

(ii) Speed of driving shaft at resonance :

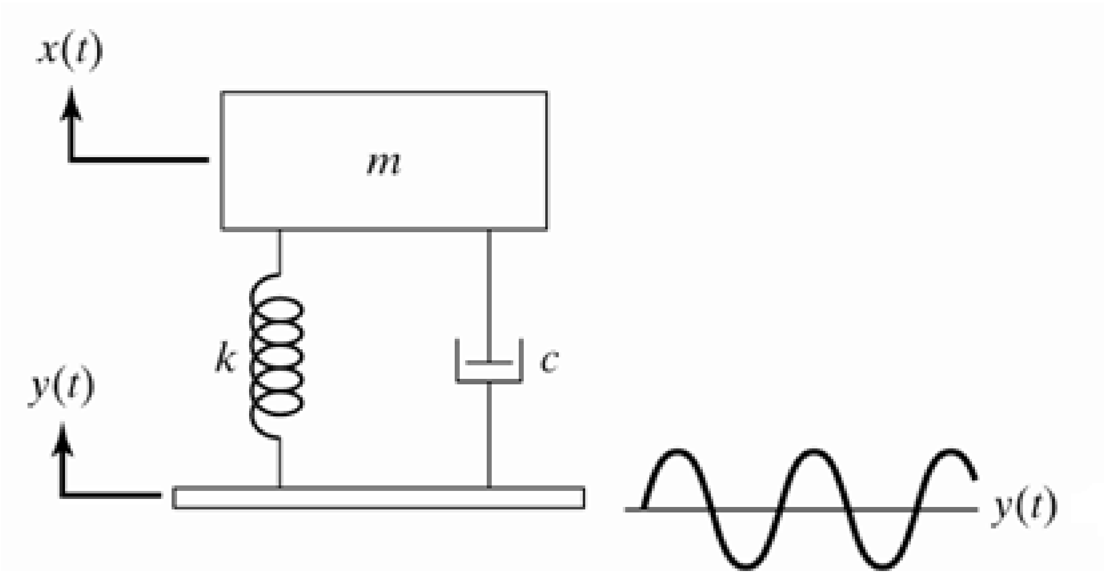
At resonance, $\omega = \omega_n = 63.93\ rad/\sec$

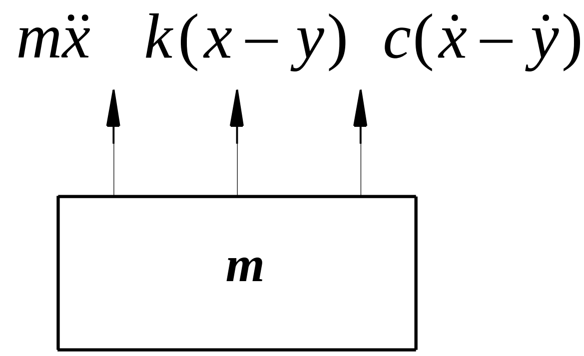
$$\Rightarrow N_{res} = \frac{60\times63.93}{2\pi} = \boldsymbol{6025\ rpm}$$

Forced Vibration due to Support Motion

- In many cases, the excitation of the system is through the support motion. ***Ex:*** *locomotives or vehicles, the wheels act as base or support for the system.*
- The wheels move vertically up & down on the road surface during the motion of the vehicle & the chassis moves relative to the wheels.
- The amplitude of motion of the chassis w.r.t road surface is known as ***absolute amplitude*** & that w.r.t wheels is known as ***relative amplitude***.
- Vibration measuring instruments are also designed on the support motion approach.

Absolute Motion (Motion transmissibility)





Absolute motion of a mass is its motion w.r.t fixed reference.Let the absolute motion of mass be ***x = A sinωt*** & the support motion be ***y = B sinωt.*** Hence the net elongation of the spring is $(x - y)$. The equation of motion can be written as;

$$m\ddot{x}+k(x-y)+c(\dot{x}-\dot{y})=0$$

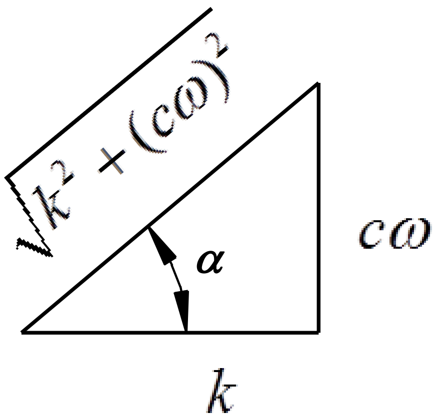
$$\Rightarrow m\ddot{x}+c\dot{x}+kx=ky+c\dot{y}\cdots\cdots(i)$$

Substituting $y = B\sin\omega t$ & $\dot{y} = \omega B\cos\omega t$ in Eqn (i),

$$m\ddot{x} + c\dot{x} + kx = B\left[k\sin\omega t + c\omega\cos\omega t\right]$$

Multiply & divide RHS by $\sqrt{k^2 + (c\omega)^2}$, we get

$$m\ddot{x} + c\dot{x} + kx = B\sqrt{k^2 + (c\omega)^2}\left[\frac{k}{\sqrt{k^2 + (c\omega)^2}}\sin\omega t + \frac{c\omega}{\sqrt{k^2 + (c\omega)^2}}\cos\omega t\right]$$



From the fig,

$$\sin\alpha = \frac{c\omega}{\sqrt{k^2 + (c\omega)^2}} \quad \& \cos\alpha = \frac{k}{\sqrt{k^2 + (c\omega)^2}}$$

Substituting in the above equation, we get

$$m\ddot{x} + c\dot{x} + kx = B\sqrt{k^2 + (c\omega)^2}\left[\cos\alpha\sin\omega t + \sin\alpha\cos\omega t\right]$$

$$\boldsymbol{m\ddot{x} + c\dot{x} + kx = B\sqrt{k^2 + (c\omega)^2}\sin(\omega t + \alpha)}$$

Comparing $m\ddot{x}+c\dot{x}+kx=B\sqrt{k^2+(c\omega)^2}\sin(\omega t+\alpha)$ with standard equation of forced vibration $m\ddot{x}+c\dot{x}+kx=F\sin\omega t$,

we see that; $F=B\sqrt{k^2+(c\omega)^2}=Bk\sqrt{1+\left(\dfrac{c\omega}{k}\right)^2}=Bk\sqrt{1+\left(2\zeta r\right)^2}$

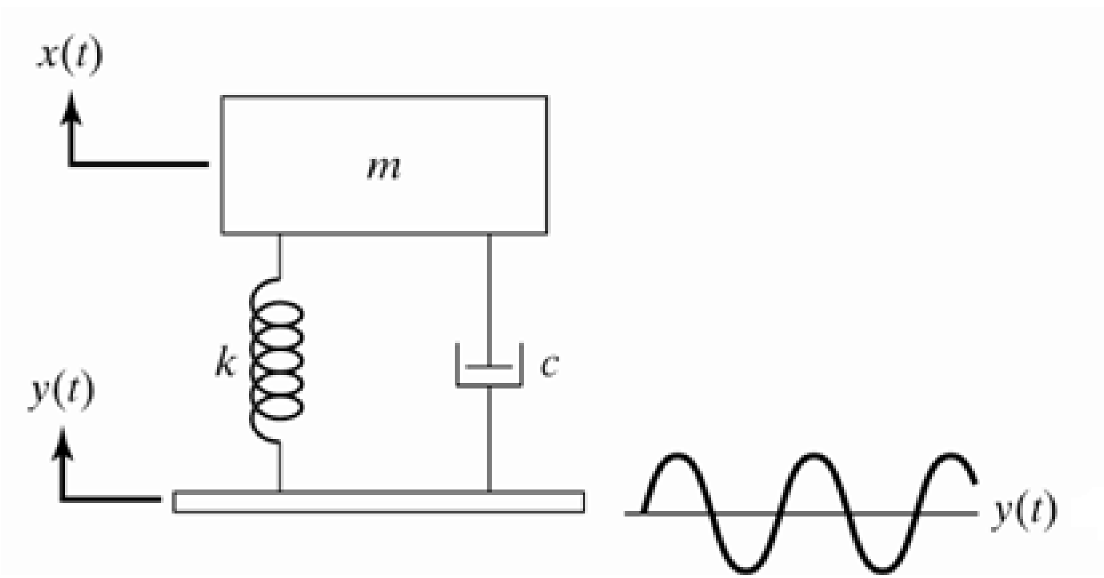
Also we know that the the steady state amplitude of forced vibration

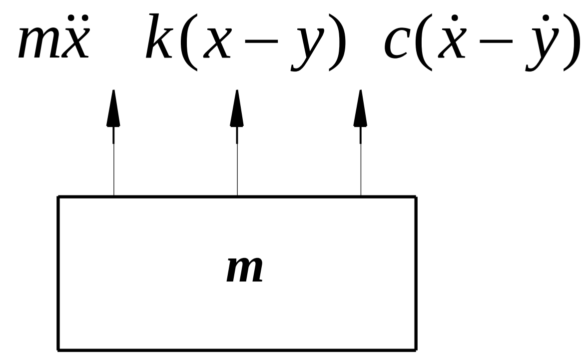
$$A=\frac{\left(\dfrac{F}{k}\right)}{\sqrt{(1-r^2)^2+\left(2\zeta r\right)^2}}=\frac{\left(\dfrac{Bk\sqrt{1+\left(2\zeta r\right)^2}}{k}\right)}{\sqrt{(1-r^2)^2+\left(2\zeta r\right)^2}}=\frac{B\sqrt{1+\left(2\zeta r\right)^2}}{\sqrt{(1-r^2)^2+\left(2\zeta r\right)^2}}$$

$\therefore$ ***Motion transmissibility*** $= \dfrac{\textbf{\textit{Amplitude of body}}}{\textbf{\textit{Amplitude of support}}}=\dfrac{A}{B}=\dfrac{\sqrt{1+\left(2\zeta r\right)^2}}{\sqrt{(1-r^2)^2+\left(2\zeta r\right)^2}}$

Phase angle between A & B $=(\phi-\alpha)=\tan^{-1}\left(\dfrac{2\zeta r}{1-r^2}\right)-\tan^{-1}2\zeta r$

Relative Motion





Relative motion of a mass is its motion w.r.t support. Let the absolute motion of mass be ***x* = *A sinωt*** & the support motion be ***y* = *B sinωt.***

Hence the relative motion is $z = (x - y) \Rightarrow x = (z + y)$

The equation of motion can be written as; $m\ddot{x} + k(x-y) + c(\dot{x}-\dot{y}) = 0$

$\Rightarrow m(\ddot{z}+\ddot{y}) + c\dot{z} + kz = 0$ ***Or*** $\boldsymbol{m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \cdots\cdots (i)}$

Substituting $y = B\sin\omega t$ & $\ddot{y} = -\omega^2 B\sin\omega t$ $in\ Eqn$ (i),

$m\ddot{z} + c\dot{z} + kz = m\omega^2 B\sin\omega t$ Comparing $m\ddot{z} + c\dot{z} + kz = m\omega^2 B\sin\omega t$ with standard equation of forced vibration $m\ddot{z} + c\dot{z} + kz = F\sin\omega t$,

we see that; $F = m\omega^2 B = \left(\frac{k}{\omega_n^2}\right)\omega^2 B = Bkr^2$ $\left(\because \omega_n^2 = \frac{k}{m} \text{ and } r = \frac{\omega}{\omega_n}\right)$

Also we know that the the steady state amplitude of forced vibration

$$Z = \frac{\left(\frac{F}{k}\right)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\left(\frac{Bkr^2}{k}\right)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{Br^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

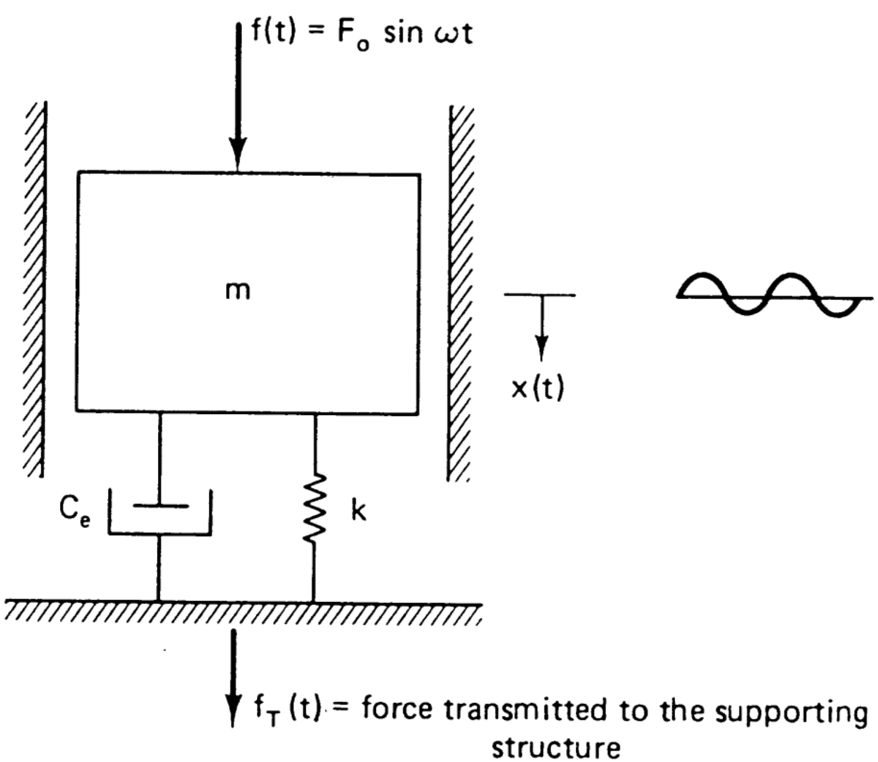
$$\therefore \frac{\textit{Relative Amplitude of body}}{\textit{Amplitude of support}} = \frac{Z}{B} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

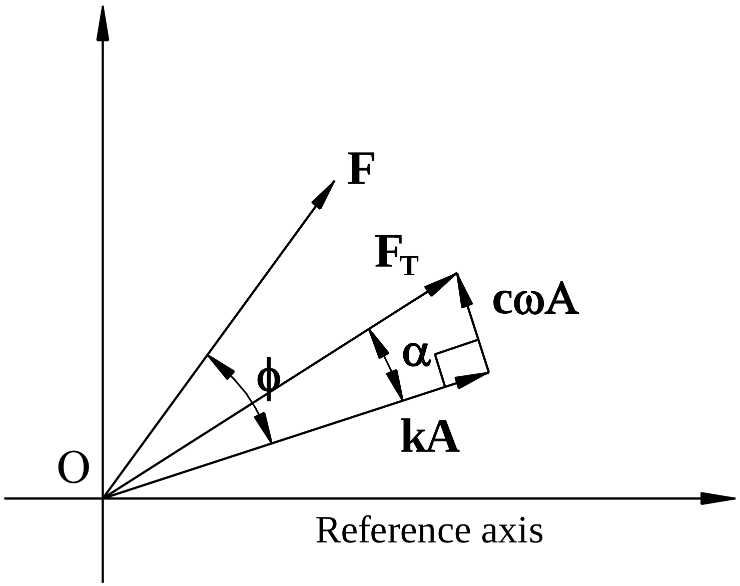
Phase angle between Z & B $= \alpha = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$

Vibration Isolation & Force Transmissibility

- Vibrations are produced in machines having unbalanced masses & these are transmitted to the foundation upon which machines are installed.
- To diminish these undesirable transmission of forces, machines are usually mounted on vibration isolating material such as ***rubber, felt cork, metallic springs***. etc. which provide stiffness & damping.
- ***Transmissibility*** (denoted by TR or ε) is the ratio of the ***force transmitted to the foundation*** to the ***force applied.***
- It is a measure of effectiveness of the vibration isolating material.

Expression for Transmissibility Ratio (TR or ε)





As the transmitted force (F_T) is the vector sum of the spring force (kA) & the damper force ($c\omega A$) which are perpendicular to each other,

$$F_T = \sqrt{(kA)^2 + (c\omega A)^2} = A\sqrt{k^2 + (c\omega)^2} = Ak\sqrt{1 + \left(\frac{c\omega}{k}\right)^2} = Ak\sqrt{1 + \left(2\zeta r\right)^2}$$

Expression for Transmissibility Ratio (TR or ε)

Also we know that amplitude of forced vibration is $A = \dfrac{\left(\dfrac{F}{k}\right)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$$\therefore F_T = \left(\frac{\left(\dfrac{F}{k}\right)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \right) k\sqrt{1+(2\zeta r)^2} = \frac{F\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

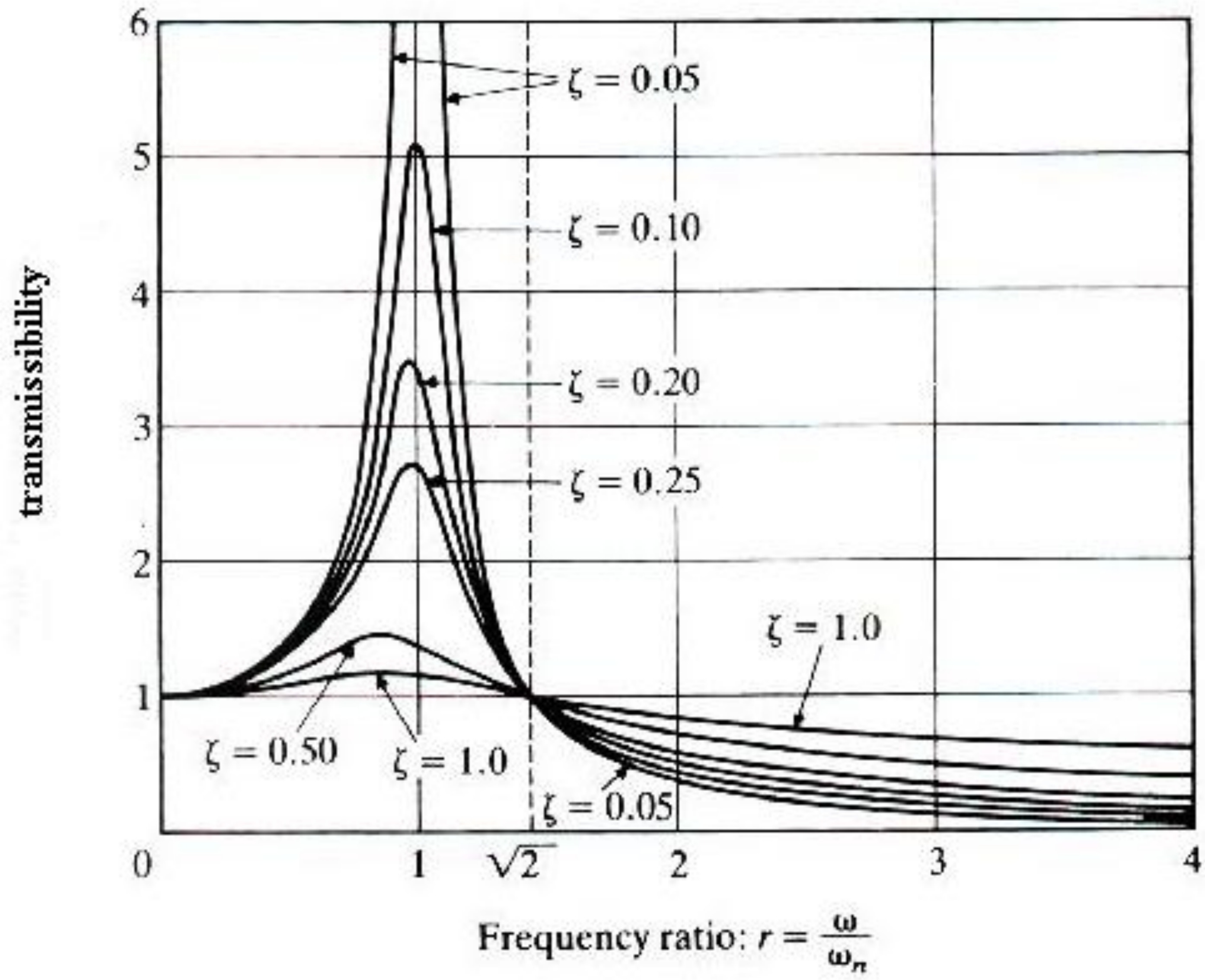
Force transmissibility ratio TR (or ε) = $\dfrac{\text{Impressed Force}}{\text{Transmitted force}}$

$$\Rightarrow \varepsilon = \frac{F_T}{F} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

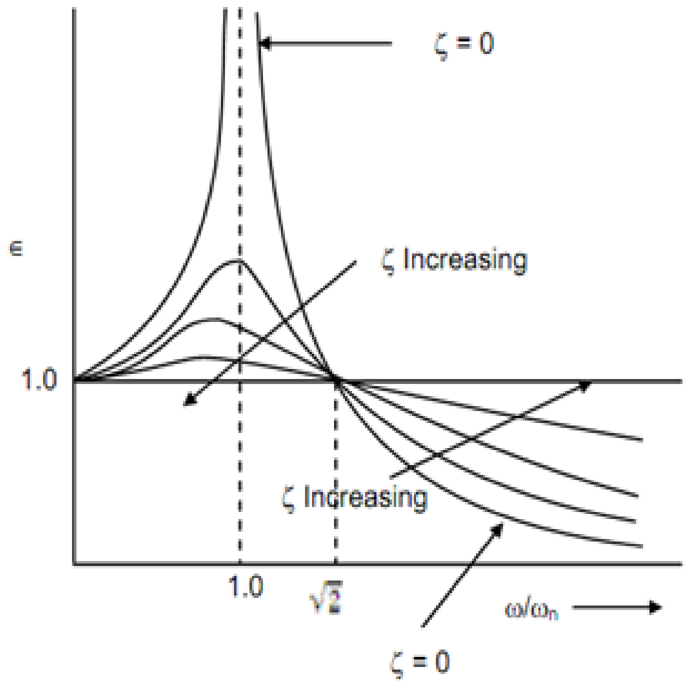
Phase angle between F & F_T = (ϕ - α) = $\tan^{-1}\left(\dfrac{2\zeta r}{1-r^2}\right) - \tan^{-1} 2\zeta r$

Note : *The expressions for motion & force transmissibility are same.*

Transmissibility vs. frequency ratio for different values of damping ratio



Transmissibility vs. frequency ratio for different values of damping ratio



(i) When $r < \sqrt{2},\ \varepsilon > 1.$ *i.e.* the transmitted force is always more than the impressed force.

(ii) When $r > \sqrt{2},\ \varepsilon < 1.$ *i.e.* the transmitted force is always less than the impressed force.

(iii) When $r = \sqrt{2},\ \varepsilon = 1.$ *i.e.* the transmitted force is always equal to the impressed force.

(iv) When $r = 1,$ the transmitted force is maximum which can be reduced by damping.

(v) When $r > \sqrt{2},$ increase in damping increases ε. Hence isolation is effective only when $r < \sqrt{2}$.

Numerical Problem 1
(Based on support excitation)

The support of a spring mass system is vibrating with an amplitude of 5 mm and a frequency of 1150 cycles per minute. If the mass is 0.9 kg and the spring has a stiffness of 1960 N/m, determine the amplitude of vibration of the mass.

What amplitude will result if a damping factor of 0.2 is included in the system?

Data : $m = 0.9\ kg,\ B = 5\ mm\ ,\ f = 1150\ cpm,\ k = 1960\, N/m$

Solution :

$$\text{Natural frequency } \omega_n = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{1960}{0.9}\right)} = \mathbf{46.67\ rad/sec}$$

$$\text{Excitation frequency } \omega = 2\pi f = 2\pi\left(\frac{1150}{60}\right) = \mathbf{120.43\ rad/sec}$$

$$\text{Frequency ratio } r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{120.43}{46.67}\right) = \mathbf{2.58}$$

(i) Without damping : ($\zeta = 0$ **)** We know that $\dfrac{A}{B} = \dfrac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$$\text{When } \zeta = 0,\ \frac{A}{B} = \frac{1}{\pm(1-r^2)},\ \text{As } r > 1,\ A = \frac{1}{(r^2-1)}$$

$$\Rightarrow \frac{A}{5} = \frac{1}{(2.58^2-1)}\ \therefore \text{Amplitude of main mass } \mathbf{A = 0.884\ mm}$$

(ii) When damping ratio $(\zeta) = 0.2:$

$$When\ \zeta = 0.2,\quad \frac{A}{B} = \frac{\sqrt{1+\left(2\times0.2\times2.58\right)^2}}{\sqrt{(1-2.58^2)^2+\left(2\times0.2\times2.58\right)^2}}$$

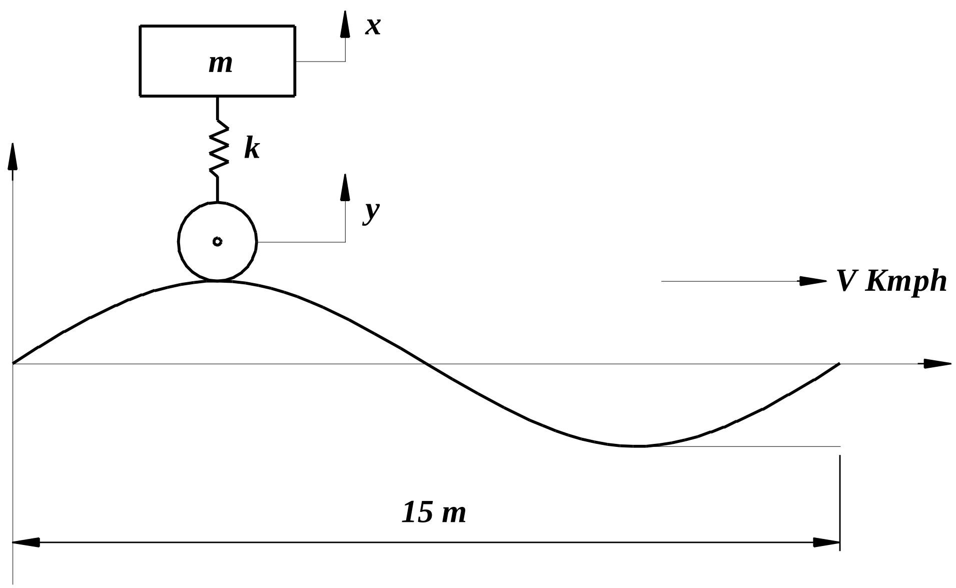
$$\frac{A}{B} = \frac{\sqrt{1+1.032^2}}{\sqrt{5.656^2+1.032^2}} = 0.25$$

$$\frac{A}{5} = 0.25 \Rightarrow A = \boldsymbol{1.25\ mm}$$

$$\therefore Amplitude\ of\ main\ mass\ \boldsymbol{A} = \boldsymbol{1.25\ mm}$$

Numerical Problem 2
(Based on support excitation)

Determine the critical speed when an automobile trailer is travelling over a road with sinusoidal profile of wavelength 15 meters and an amplitude of 75 mm. The springs of the automobile are compressed 0.125 m under its own weight. Also determine the amplitude of vibration at 50 kmph.



Data : $\Delta = 0.125\ m,\ B = 75\ mm\ ,\ v = 50\ kmph,$ Wave length $\lambda = 15\ m$

Solution :

(i) Critical speed of trailer :

$$Natural\ frequency\ \omega_n = \sqrt{\left(\frac{g}{\Delta}\right)} = \sqrt{\left(\frac{9.81}{0.125}\right)} = \mathbf{8.86\ rad\ /\ sec}$$

$$\text{For critical speed, } \omega = \omega_n = 8.86 \Rightarrow f_n = \left(\frac{2\pi}{\omega_n}\right) = \mathbf{1.41\ Hz}$$

We know that **Linear velocity = frequency × wave length**

$$i.e.\, v = f_n \times \lambda = 1.41 \times 15 = \mathbf{21.15\ m\ /\ sec}$$

$$To\ convert\ m\ /\ sec\ into\ kmph, multiply\ by \left(\frac{3600}{1000}\right)$$

$$\Rightarrow Critical\ \ Velocity\ of\ \ the\ automobile = 21.15 \times \left(\frac{3600}{1000}\right) = \mathbf{76.14\ kmph}$$

(ii) Amplitude of vibration at 50 kmph :

$$Velocity\ in\ m/\sec\ \ v = 50\times\left(\frac{1000}{3600}\right) = \mathbf{13.89\ m\ /\ sec}$$

$$We\ know\ that\ v = f\times\lambda \Rightarrow 13.89 = f\times 15\ \therefore\ f = \mathbf{0.926\ Hz}$$

$$\Rightarrow \omega = 2\pi f = 5.82\ rad/\sec.\ \text{Also}\ \omega_n = 46.67\ rad/\sec$$

$$Frequency\ ratio\ r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{5.82}{8.86}\right) = \mathbf{0.657}$$

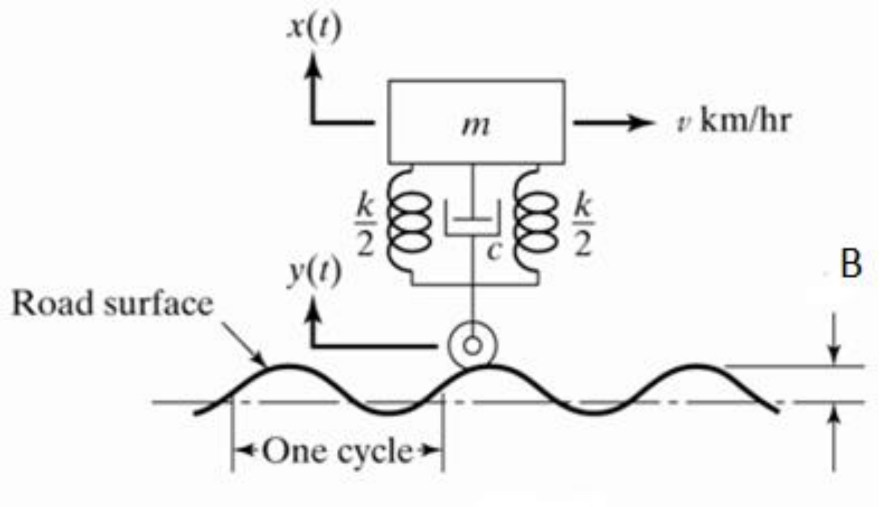
$$When\ \zeta = 0,\ \ \frac{A}{B} = \frac{1}{\pm(1-r^2)},\ As\ r<1,\ \ \frac{A}{B} = \frac{1}{(1-r^2)}$$

$$\Rightarrow \frac{A}{75} = \frac{1}{(1-0.657^2)}\ \therefore\ Amplitude\ of\ trailer\ \mathbf{A = 132\ mm}$$

Numerical Problem 3
(Based on support excitation)

A trailer has 1000 Kg mass when fully loaded and 250 Kg when empty. The suspension has a stiffness of 350 kN/m. The damping factor is 0.5. The speed of the trailer is 100 Km/hr. The road varies sinusoidally with a wave length of 5 m. Determine the amplitude ratio of the trailer:

1. When fully loaded.
2. When empty.



Data :

$m = 1000\ kg\ (full\ \ load)\ \ \&\ \ m = 250kg\ \ (Empty), v = 100\ kmph,\ \ \zeta = 0.5$

$Wavelength\ of\ \ road\ \ suface\ \lambda = 5\ m,\ \ k = 300\ KN/m = 300 \times 10^3\ N/m$

Solution :

$Velocity\ of\ \ the\ vehicle\ v = 100\ kmph = 100 \times \left(\frac{1000}{3600}\right) = \mathbf{27.78\ m / sec}$

$We\ know\ that$ ***Linear velocity (v) = frequency (f)*** $\times$ ***wave length (*** λ ***)***

$i.e.\ 27.78 = f \times 5 \Rightarrow f = \mathbf{5.556\ Hz}$

$\therefore \omega = 2\pi f = 2\pi \times 5.556 = \mathbf{34.91\ rad / sec}$

(i) Amplitude ratio when vehicle is fully loaded : (m = 1000 kg)

$$\text{Natural frequency } \omega_n = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{350\times10^3}{1000}\right)} = \mathbf{18.71\ rad / sec}$$

$$\text{Ratio of frequencies } r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{34.91}{18.71}\right) = \mathbf{1.87}$$

$$\frac{A}{B} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{\sqrt{1+(2\times0.5\times1.87)^2}}{\sqrt{(1-1.87^2)^2+(2\times0.5\times1.87)^2}} = \mathbf{0.68}$$

(ii) Amplitude ratio when vehicle is empty : (m = 250 kg)

$$\text{Natural frequency } \omega_n = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{350\times10^3}{250}\right)} = \mathbf{37.42\ rad / sec}$$

$$\text{Ratio of frequencies } r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{34.91}{37.42}\right) = \mathbf{0.933}$$

$$\frac{A}{B} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{\sqrt{1+(2\times0.5\times0.933)^2}}{\sqrt{(1-0.933^2)^2+(2\times0.5\times0.933)^2}} = \mathbf{1.452}$$

Numerical Problem 3
(Both absolute motion & relative motion included)

A TV set of 25 kg mass must be isolated from a machine vibrating with an amplitude 0f 0.1 mm at 1000 rpm. The TV set is mounted on five isolators each having 30 KN/m and a damping constant of 400 N-sec/m. Determine;

(i) Amplitude of vibration of TV set.

(ii) Dynamic load on each isolator due to vibration.

Data : $m = 25\ kg,\ N = 1000\ rpm\ ,\ B\ (Support\ amplitude) = 0.1\ mm$

$$N = 1000\ rpm, c_{eq} = (400\ \times 5) = 2000\ N - \mathrm{sec}/\ m$$

$$k_{eq} = (5 \times 30) = 150 KN\ /\ m = 150 \times 10^{3}\ N\ /\ m$$

Solution :

$$Natural\ frequency\ \omega_n = \sqrt{\left(\frac{k_{eq}}{m}\right)} = \sqrt{\left(\frac{150 \times 10^{3}}{25}\right)} = \boldsymbol{77.46\ rad\ /\ sec}$$

$$Excitation\ frequency,\ \omega = \left(\frac{2\pi N}{60}\right) = \left(\frac{2\pi \times 1000}{60}\right) = \boldsymbol{104.72\ rad\ /\ sec}$$

$$Ratio\ of\ \ frequencie,\ r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{104.72}{77.46}\right) = \boldsymbol{1.352}$$

$$Damping\ ratio\ of\ \zeta = \left(\frac{c}{c_c}\right) = \left(\frac{c}{2m\omega_n}\right) = \left(\frac{2000}{2 \times 25 \times 77.46}\right) = \boldsymbol{0.5164}$$

(i) Amplitude of vibration of T V set : $\dfrac{A}{B} = \dfrac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$

$$When\ \zeta = 0.5164,\quad \frac{A}{B} = \frac{\sqrt{1+(2\times 0.5164\times 1.352)^2}}{\sqrt{(1-1.352^2)^2+(2\times 0.5164\times 1.352)^2}}$$

$$\Rightarrow \frac{A}{0.1} = \frac{1.7175}{1.623} = 1.058 \therefore Amplitude\ of\ main\ mass\ \mathbf{A} = \boldsymbol{0.106\ mm}$$

(ii) Dynamic load on isolators : *W.K.T Relative amplitude ratio*

$$\frac{Z}{B} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{1.352^2}{\sqrt{(1-1.352^2)^2+(2\times 0.5164\times 1.352)^2}}$$

$$\frac{Z}{B} = 1.126 \Rightarrow Z = 1.126\times 0.1 = 0.1126\ mm = \boldsymbol{0.1126\times 10^{-3}\ mm}$$

$$Dynamic\ load\ on\ each\ isolator\ F_D = Z\sqrt{k^2+(c\omega)^2}$$

$$F_D = 0.1126\times 10^{-3}\sqrt{(30000)^2+(400\times 104.72)^2} = \boldsymbol{5.8N}$$

Numerical Problem 1
Force Transmissibility problems

A reciprocating machine of mass 75 Kg is mounted on springs of stiffness 11.76×10^5 N/m and a damper of damping factor 0.2. The slider of mass 2 Kg within the machine has a reciprocating motion with a stroke of 0.08 m. The speed is 3000 rpm. Assuming the motion of the piston to be harmonic.

1. Amplitude of vibration of the machine.
2. Transmissibility ratio.
3. Force transmitted to the foundation.
4. Is vibration isolation achieved? If so how?

Data : $M = 75\ kg, m = 2\ kg,\ N = 3000\ rpm,\ \zeta = 0.2, k = 11.76\times10^5\ N/m$

Stroke of piston $= 0.08\ m \Rightarrow$ crank radius $e = 0.04\ m$

Solution :

$$Natural\ frequency\ \omega_n = \sqrt{\left(\frac{k}{M}\right)} = \sqrt{\left(\frac{11.76\times10^5}{75}\right)} = \boldsymbol{125.22\ rad/sec}$$

$$Forcing\ frequency\ \omega = \frac{2\pi N}{60} = \frac{2\pi\times3000}{60} = \boldsymbol{314.16\ rad/sec}$$

$$Frequency\ ratio\ r = \left(\frac{\omega}{\omega_n}\right) = \left(\frac{314.16}{125.22}\right) = \boldsymbol{2.51}$$

$$Unbalanced\ \ force\ F = m\omega^2 e = 2\times(314.16)^2\times0.04 = \boldsymbol{7896\ N}$$

(i) Amplitude of vibration : $A = \dfrac{\dfrac{F}{k}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$$A = \frac{\left(\dfrac{7896}{11.76\times 10^5}\right)}{\sqrt{(1-2.51^2)^2 + (2\times 0.2\times 2.51)^2}} = \frac{6.714\times 10^{-3}}{\sqrt{28.09+1.008}} = 1.245\times 10^{-3}\ m$$

$\therefore$ *Amplitude of vibration of machine* $\mathbf{A = 1.245\ mm}$

(ii) Transmissibility ratio :

$$TR = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\sqrt{1+(2\times 0.2\times 2.51)^2}}{\sqrt{(1-2.51^2)^2 + (2\times 0.2\times 2.51)^2}}$$

$$\therefore TR \text{ (or } \varepsilon) = \frac{1.417}{5.394} = \boldsymbol{0.2627}$$

(iii) Force transmitted to the foundation :

$$As\ TR = \frac{Force\ \ transmitted}{Force\ impressed} = \frac{F_T}{F} \Rightarrow 0.2627 = \frac{F_T}{7896}$$

$$\therefore Force\ transmitted\ to\ the\ \ foundation\ F_T = \boldsymbol{2074.3\ N}$$

(iv) Check for vibration isolation :

As the force transmitted is less than force impressed &

$$r = 2.51 > \sqrt{2}, vibration\ \ isolation\ \ is\ \ achieved.$$

Numerical Problem 2
Force Transmissibility problems

A refrigerator of mass 35 Kg operating at 480 rpm is supported on 3 springs. If only 10% of the shaking force is to be transmitted to the foundation what should be the value of spring rate *K*?

Data : $TR = 10\% = 0.1,\ m = 35\ kg,\ N = 480\ rpm,\ No\ of\ springs\ = 3$

$\zeta = 0\ (As\ no\ damper\ present)$

Solution : $Forcing\ frequency\ \omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 480}{60} = \mathbf{50.267\ rad / sec}$

$$We\ know\ that\ when\ \zeta = 0,\ TR = \frac{1}{\pm(1-r^2)} \Rightarrow 0.1 = \frac{1}{\pm(1-r^2)}$$

$$i.e.\ \pm(1-r^2)=\left(\frac{1}{0.1}\right)=10$$

Taking +ve sign, $r^2=-9 \Rightarrow\ r=\sqrt{-9}$ (Not possible)

Taking +ve sign, $r^2=11 \Rightarrow\ r=\sqrt{11}=\boldsymbol{3.32}$

$$But\ frequency\ ratio\ r=\left(\frac{\omega}{\omega_n}\right)\Rightarrow 3.32=\left(\frac{50.267}{\omega_n}\right)\therefore \omega_n=\boldsymbol{15.16\ rad\ /\ sec}$$

$$Combined\ stiffness\ of\ springs\ k_{eq}=m\omega_n^2=35\times(15.16)^2=\boldsymbol{8040\ N\ /\ m}$$

As there are three springs, $k=\left(\frac{k_{eq}}{3}\right)=\left(\frac{8040}{3}\right)$

$\therefore$ ***Stiffness of each spring is 2680 N / m***

Numerical Problem 3 (Force Transmissibility problems)

A machine supported symmetrically on four springs has a mass of 80 Kgs. The mass of the reciprocating mass is 2.2 Kgs which move through a vertical stroke of 100 mm with SHM. Neglecting damping, determine the combined stiffness of the springs so that the force transmitted to the foundation is 1/20th of the impressed force. the machine crank shaft rotates at 800 rpm.

If, under actual working conditions, the damping reduces the amplitudes of successive vibrations by 30%, find

(a) The force transmitted to the foundation at 800 rpm

(b) The force transmitted to the foundation at resonance.

(c) The amplitude of vibrations at resonance.

Data : $M = 80\ kg,\ m = 2.2\ kg,\ N = 800\ rpm, \Delta = 2.4 = 2.4\times 10^{-3} m$

Stroke of piston =100 mm $\Rightarrow$ crank radius $e = 50\ mm = 0.05\ m$

$$TR = \left(\frac{1}{20}\right) = 0.05,\ x_{n+1} = (1-0.3)x_n \Rightarrow \left(\frac{x_n}{x_{n+1}}\right) = \left(\frac{1}{0.7}\right) = 1.43$$

Solution :

$$Forcing\ frequency\ \omega = \frac{2\pi N}{60} = \frac{2\pi\times 800}{60} = \boldsymbol{83.78\ rad\ /\ sec}$$

$$Logarithimic\ decrement\ \ \delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \ln(1.43) = \boldsymbol{0.358}$$

$$Damping\ factor\ \zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{0.358}{\sqrt{4\pi^2+0.358^2}} = \boldsymbol{0.057}$$

$$Unbalanced\ \ force\ F = m\omega^2 e = 2.2\times(83.78)^2\times 0.05 = \boldsymbol{772\ N}$$

(i) Neglecting damping : (ζ = 0)

$$We\ know\ that\ when\ \zeta = 0,\ TR = \frac{1}{\pm(1-r^2)} \Rightarrow 0.05 = \frac{1}{\pm(1-r^2)}$$

$$i.e.\ \pm(1-r^2) = \left(\frac{1}{0.05}\right) = 20$$

$$\text{Taking +ve sign, } r^2 = -19 \Rightarrow\ r = \sqrt{-19}\ \ \text{(Not possible)}$$

$$\text{Taking} - \text{ve sign, } r^2 = 21 \Rightarrow\ r = \sqrt{21} = \boldsymbol{4.58}$$

$$But\ frequency\ ratio\ r = \left(\frac{\omega}{\omega_n}\right) \Rightarrow 4.58 = \left(\frac{83.78}{\omega_n}\right) \therefore \omega_n = \boldsymbol{18.28\ rad\ /\ sec}$$

$$Combined\ stiffness\ of\ springs\ k_{eq} = M\omega_n^2 = 80 \times (18.28)^2 = \boldsymbol{26740\ N\ /\ m}$$

(ii) Force transmitted at 800 rpm with damping (ζ = 0.057) :

$$TR = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{\sqrt{1+(2\times 0.057\times 4.58)^2}}{\sqrt{(1-4.58^2)^2+(2\times 0.057\times 4.58)^2}}$$

$$\therefore TR \text{ (or } \varepsilon) = \boldsymbol{0.0564} = \frac{F_T}{F}$$

$\therefore$ Force transmitted to foundation at 800 rpm

$$F_T = 0.0564\times F = 0.0564\times 772 = \boldsymbol{43.55\ N}$$

(iii) Force transmitted at resonance with damping (ζ = 0.057) :

$$resonance,\ \omega = \omega_n \Rightarrow r = 1,$$

$$\therefore TR = \frac{\sqrt{1+(2\zeta)^2}}{2\zeta} = \frac{\sqrt{1+(2\times 0.057)^2}}{2\times 0.057} = \boldsymbol{8.83}$$

$$Also,\ at\ resonance,\ F_{res} = m\omega_n^2 e = 2.2\times(18.28)^2\times 0.05 = \boldsymbol{36.76\ N}$$

$i.e.$ Force transmitted t $F_T = 8.83\times F_{res} = 8.83\times 36.76 = \boldsymbol{325\ N}$

(iv) Amplitude of vibration at resonance : $(r=1)$

$$A=\frac{\frac{F}{k}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\Rightarrow A_{res}=\frac{\left(\frac{F_{res}}{k}\right)}{2\zeta}=\frac{\left(\frac{36.76}{26740}\right)}{2\times 0.057}=\boldsymbol{0.012\ m}$$