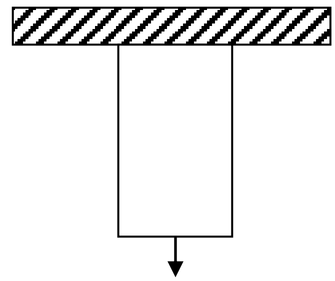
FEM
Module One Problems
By V Balaraj

1. A rectangular bar is subjected to an axial load P as shown in Figure. Determine the expression of PE functional and determine the extreme value of the PE for the following data E= 200 GPa, P = 3kN, L=100 mm, b=width = 20 mm and t = 10 mm .



Solution

PE= SE + WP

$$SE = \frac{1}{2}\sigma \ \in V = \frac{1}{2}\sigma \ \in A\, L$$

$$\in = \frac{u}{L} \qquad and\ \sigma = E \in = \frac{Eu}{L}$$

$$SE = \frac{1}{2}\frac{Eu^2AL}{L^2} = \frac{1}{2}\frac{Eu^2A}{L}$$

WP = - p u

$$\pi = \frac{1}{2}\frac{Eu^2A}{L} - p\,u$$

Apply minimum PE principle

$$\frac{\partial \pi}{\partial u} = \frac{Eu \quad A}{L} - p = 0$$

$$u = \frac{3000\,x100}{10x20x200x10^3} \qquad = 7.5\,x10^{-3}mm$$

$$\pi = \frac{1}{2}\frac{Eu^2A}{L} - p\,u = -11.25\ N - mm$$

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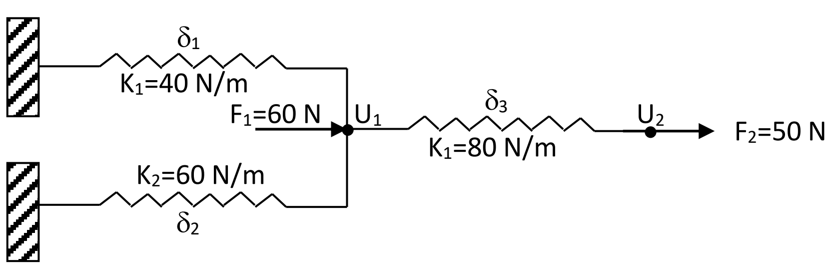
2. Using the principle of minimum potential energy determine the displacement at the nodes for a given spring system shown in figure.



PE = SE + WP

$$SE = \frac{1}{2}k_1\delta_1^2 + \frac{1}{2}k_2\delta_2^2 + \frac{1}{2}k_3\delta_3^2$$

$$SE = \frac{1}{2}k_1u_1^2 + \frac{1}{2}k_2u_2^2 + \frac{1}{2}k_3(u_2 - u_1)^2$$

$$WP = -F_1u_1 - F_2u_2$$

$$\pi = \frac{1}{2}k_1u_1^2 + \frac{1}{2}k_2u_1^2 + \frac{1}{2}k_3(u_2 - u_1)^2 - F_1u_1 - F_2u_2$$

$$\frac{\vartheta\pi}{\vartheta u_1} = k_1u_1 + k_2u_1 - k_3(u_2 - u_1) - F_1 = 0$$

$$(k_1 + k_2 + k_3)u_1 - k_3u_2 = F_1$$

$$180\,u_1 - 80u_2 = 60 \qquad 01$$

$$\frac{\vartheta\pi}{\vartheta u_2} = -k_3u_1 + k_3u_2 - F_2 = 0$$

$$-k_3u_1 + k_3u_2 = F_2$$

$$-80\,u_1 + 80u_2 = 50 \qquad 02$$

U_1 = 1.1 m and U_2 = 1.725 m

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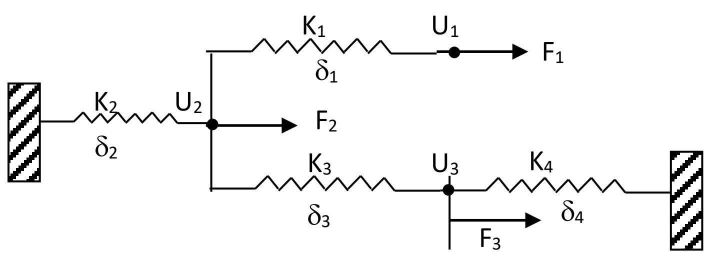
3. Using the principle of minimum potential energy determine the displacement at the nodes for a given spring system shown in figure.



PE = SE + WP

$$SE = \frac{1}{2}k_1\delta_1^2 + \frac{1}{2}k_2\delta_2^2 + \frac{1}{2}k_3\delta_3^2 + \frac{1}{2}k_3\delta_4^2$$

$$SE = \frac{1}{2}k_1(u_1 - u_2)^2 + \frac{1}{2}k_2u_2^2 + \frac{1}{2}k_3(u_3 - u_2)^2 + \frac{1}{2}k_4u_3^2$$

$$WP = -F_1u_1 - F_2u_2 - F_3u_3$$

$$\pi = \frac{1}{2}k_1(u_1 - u_2)^2 + \frac{1}{2}k_2u_2^2 + \frac{1}{2}k_3(u_3 - u_2)^2 + \frac{1}{2}k_4u_3^2 - F_1u_1 - F_2u_2 - F_3u_3$$

$$\frac{\vartheta\pi}{\vartheta u_1} = k_1(u_1 - u_2) \quad - F_1 = 0$$

$$k_1u_1 - k_1u_2 = F_1 \qquad \mathbf{01}$$

$$\frac{\vartheta\pi}{\vartheta u_2} = -k_1(u_1 - u_2) \quad + k_2u_2 \; - k_3(u_3 - u_2) \quad - F_2 = 0$$

$$-k_1u_1 + k_1u_2 + k_2u_2 + k_3u_2 - k_3u_3 = F_2$$

$$-k_1u_1 + (k_1 + k_2 + k_3)u_2 - k_3u_3 = F_2 \qquad \mathbf{02}$$

$$\frac{\vartheta\pi}{\vartheta u_3} = k_3(u_3 - u_2) \quad + k_4u_3 \; - F_3 = 0$$

$$-k_3u_2 + (k_3 + k_4)u_3 = F_3 \qquad \mathbf{03}$$

From Equation 1, 2 and 3

$$\begin{bmatrix} k_1 & -k1 & 0 \\ -k_1 & k_1 + k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

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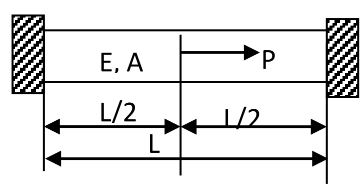
4. Figure shows a bar fixed at both ends subjected to an axial load. Determine the displacement at the loading point and the corresponding stress using Rayleigh- Ritz method.



Step 1: Formulation of PE functions

$\prod$ = SE + WP

$$\prod = \frac{EA}{2}\int_0^l \left(\frac{\partial u}{\partial x}\right)^2 dx - p\ u_m \qquad 01$$

Step 2: Selection of Displacement Model

$$u = a_0 + a_1x + a_2x^2 \qquad 02$$

BC-1 x = 0, u=0 then a_0=0

BC-2 x = L, u = 0 → $0 = a_1L + a_2L^2$

$a_1 = -a_2L$

$$u = -a_2Lx + a_2x^2 \rightarrow u = a_2(x^2 - Lx) \qquad 03$$

$$\frac{\partial u}{\partial x} = a_2(2x - L)$$

$$\left(\frac{\partial u}{\partial x}\right)^2 = a_2^2(4x^2 + L^2 - 4Lx) \qquad 04$$

$$x = \frac{L}{2} \rightarrow\rightarrow\rightarrow u = u_m = a_2\left(\frac{L^2}{4} - \frac{L^2}{2}\right) \rightarrow\rightarrow u_m = -a_2\frac{L^2}{4} \qquad 05$$

Step 3: Substitute in PE

$$\prod = \frac{\mathrm{EA}}{2}\int_0^l a_2^2(4x^2 + L^2 - 4Lx)\mathrm{dx} + \mathrm{p}\ a_2\frac{L^2}{4}$$

$$\prod = \frac{\mathrm{E\,a_2^2L^3}}{6} + \frac{\mathrm{p\,a_2L^2}}{4} \qquad 06$$

Step 4: Applied Minimum Potential Energy

$$\frac{\partial\prod}{\partial \mathrm{x}} = \frac{\mathrm{2E\,a_2\,L^3}}{6} + \frac{\mathrm{p\,L^2}}{4} = 0$$

$$a_2 = -\frac{3p}{4EAL} \qquad 07$$

Substitute in Equation 03

$$u = -\frac{3p}{4EAL}(x^2 - Lx) \qquad 08$$

$$x = \frac{L}{2}\ u = u_m = -\frac{3pL}{16EA}$$

Position (x)	Strain= $\frac{\partial u}{\partial x} = -\frac{3p}{4EAL}(2x - L)$	Stress = $-\frac{3p}{4AL}(2x - L)$
x = 0	$\frac{3p}{4EA}$	$\frac{3p}{4A}$
x=L/2	0	0
x = L	$-\frac{3p}{4EA}$	$-\frac{3p}{4A}$

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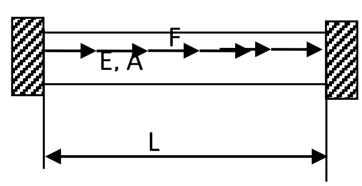
5. Figure shows a bar fixed at both ends subjected to continuous axial load. Determine the displacement at the loading point and the corresponding stress using Rayleigh- Ritz method.



Step 1: Formulation of PE functions

$\prod$ = SE + WP

$$\prod = \frac{\mathrm{EA}}{2}\int_0^l\left(\frac{\partial \mathrm{u}}{\partial \mathrm{x}}\right)^2 \mathrm{dx} - \int_0^L \mathrm{Fudx} \qquad 01$$

Step 2: Selection of Displacement Model

$u = a_0 + a_1 x + a_2 x^2$ 02

BC-1 x = 0, u=0 then a_0=0

BC-2 x = L, u = 0 → $0 = a_1 L + a_2 L^2$

$a_1 = -a_2 L$

$u = -a_2 Lx + a_2 x^2 \rightarrow u = a_2(x^2 - Lx)$ 03

$\frac{\partial u}{\partial x} = a_2(2x - L)$

$\left(\frac{\partial u}{\partial x}\right)^2 = a_2^2(4x^2 + L^2 - 4Lx)$ 04

$\int_0^L \mathrm{F}a_2(x^2 - Lx)\mathrm{dx} = -\frac{\mathrm{Fa_2L^3}}{6}$ 05

Step 3: Substitute in PE

$\prod = \frac{\mathrm{EA}}{2}\int_0^l a_2^2(4x^2 + L^2 - 4Lx)\mathrm{dx} + \frac{\mathrm{Fa_2L^3}}{6}$

$\prod = \frac{\mathrm{E\,a_2^2L^3}}{6} + \frac{\mathrm{F\,a_2L^3}}{6}$ 06

Step 4: Applied Minimum Potential Energy

$\frac{\partial\prod}{\partial \mathrm{x}} = \frac{2\mathrm{E\,a_2\,L^3}}{6} + \frac{\mathrm{F\,L^3}}{6} = 0$

$a_2 = -\frac{F}{2EA}$ 07

Substitute in Equation 03

$u = -\frac{F}{2EA}(x^2 - Lx)$ 08

$x = \frac{L}{2}\; u = u_m = -\frac{3FL^2}{8EA}$

Position (x)	Strain= $\frac{\partial u}{\partial x} = -\frac{F}{2EA}(2x - L)$	Stress = $-\frac{F}{2A}(2x - L)$
x = 0	$\frac{FL}{2EA}$	$\frac{FL}{2A}$
x=L/2	0	0
x = L	$-\frac{FL}{2EA}$	$-\frac{FL}{2A}$

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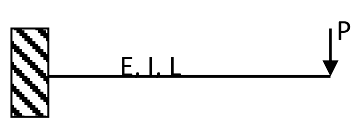
6. A cantilever beam of span L is subjected to a point load at free end. Derive an equation for the deflection at free end by using Rayleigh- Ritz method. Assume polynomial function



Step 1: Formulation of PE functions

$\prod$ = SE + WP

$\prod = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx - py_m$ 01

Step 2: Selection of Displacement Model

$y = a_0 + a_1x + a_2x^2 + a_3x^3$ 02

BC-1 x = 0,y=0 then a_0=0

BC-2 x = 0, $\frac{\partial Y}{\partial x} = 0$ then a_1=0

BC-3 x=L $\frac{\partial^2 Y}{\partial x^2} = 0 \quad 0 = 2a_2 + 6a_3L$

$a_2 = -3a_3L$

$y = -3a_3Lx^2 + a_3x^3 \rightarrow y = a_3(x^3 - 3Lx^2)$ 03

$\frac{\partial y}{\partial x} = a_3(3x^2 - 6Lx)$

$\frac{\partial^2 y}{\partial x^2} = 6a_3(x \quad - L)$

$\left(\frac{\partial^2 y}{\partial x^2}\right)^2 = 36a_3^2(x^2 + L^2 - 2Lx)$ 04

$x = L \rightarrow\rightarrow\rightarrow y = y_m = a_3(L^3 - 3L^3) \rightarrow\rightarrow Y_m = -2a_3L^3$ 05

Step 3: Substitute in PE

$\prod = \frac{EI}{2}\int_0^l 36a_3^2(x^2 + L^2 - 2Lx) + 2a_3L^3P$

$\prod = 6EIa_3^2L^3 + 2a_3L^3P$ 06

Step 4: Applied Minimum Potential Energy

$\frac{\partial\prod}{\partial x} = 12EIa_3L^3 + 2L^3P = 0$

$a_3 = -\frac{P}{6EI}$ 07

Substitute in Equation 03

$y = -\frac{P}{6EI}(x^3 - 3Lx^2)$ 08

$y_m = \frac{PL^3}{3EI}$

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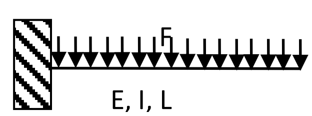
7. A cantilever beam of span L is subjected to a uniformly distributed load. Derive an equation for the deflection at free end by using Rayleigh- Ritz method. Assume



polynomial function

Step 1: Formulation of PE functions

$\prod = SE + WP$

$\prod = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx - \int_0^L Fydx$ 01

Step 2: Selection of Displacement Model

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ 02

BC-1 x = 0, y=0 then a_0=0

BC-2 x = 0, $\frac{\partial Y}{\partial x} = 0$ then a_1=0

BC-3 x=L $\frac{\partial^2 Y}{\partial x^2} = 0 \quad 0 = 2a_2 + 6a_3 L$

$a_2 = -3a_3 L$

$y = -3a_3 Lx^2 + a_3 x^3 \rightarrow y = a_3(x^3 - 3Lx^2)$ 03

$\frac{\partial y}{\partial x} = a_3(3x^2 - 6Lx)$

$\frac{\partial^2 y}{\partial x^2} = 6a_3(x \quad - L)$

$\left(\frac{\partial^2 y}{\partial x^2}\right)^2 = 36a_3^2(x^2 + L^2 - 2Lx)$ 04

$\int_0^L Fa_3(x^3 - 3Lx^2)dx = -\frac{3Fa_3L^4}{4}$ 05

Sep 3: Substitute in PE

$\prod = \frac{EI}{2}\int_0^l 36a_3^2(x^2 + L^2 - 2Lx) + \frac{3Fa_3L^4}{4}$

$\prod = 6EIa_3^2L^3 + \frac{3Fa_3L^4}{4}$ 06

Step 4: Applied Minimum Potential Energy

$\frac{\partial \prod}{\partial x} = 12EIa_3L^3 + \frac{3FL^4}{4} = 0$

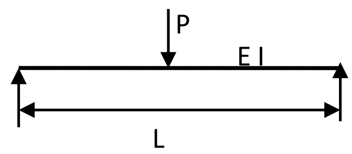
$a_3 = -\frac{FL}{16EI}$ 07

Substitute in Equation 03

$y = -\frac{FL}{16EI}(x^3 - 3Lx^2)$ 08

$y_m = \frac{FL^4}{8EI}$

8. Simply supported beam subjected to point load at the centre. Derive an equation for maximum deflection using trigonometrical function by Rayleigh Ritz method



Step 1: Formulation of PE functions

Π = SE + WP

$$\Pi = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx - py_m \qquad 01$$

Step 2: Selection of Displacement Model

$$y = C \sin\frac{\pi x}{L} \qquad 02$$

BC-1 x = 0 then y =0

BC-2 x=L then y =0

BC-3 x = L/2 then $\frac{\partial Y}{\partial x} = 0$

$$y = C \sin\frac{\pi x}{L} \qquad 03$$

$$\frac{\partial^2 y}{\partial x^2} = -C\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}$$

$$\left(\frac{\partial^2 y}{\partial x^2}\right)^2 = C^2\frac{\pi^4}{L^4}\sin\frac{\pi x}{L} \qquad 04$$

$$x = \frac{L}{2} \rightarrow\rightarrow\rightarrow y = y_m = C \qquad 05$$

Step 3: Substitute in PE

$$\Pi = \frac{EI}{2}\int_0^l C^2\frac{\pi^4}{L^4}\sin\frac{\pi x}{L} - Cp$$

$$\Pi = \frac{C^2 EI\pi^4}{4L^3} - pC \qquad 06$$

Step 4: Applied Minimum Potential Energy

$$\frac{\partial \Pi}{\partial x} = \frac{2C \ \ EI\pi^4}{4L^3} - p = 0$$

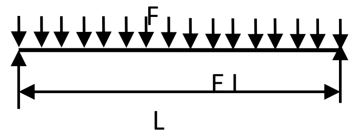
$$C = \frac{2PL^3}{EI\pi^4} \qquad 07$$

Substitute in Equation 03

$$y = \frac{2PL^3}{EI\pi^4}\sin\frac{\pi x}{L} \qquad 08$$

$$Y = \frac{2PL^3}{EI\pi^4}$$

9. Simply supported beam subjected uniform distributed load on the beam. Derive an equation for maximum deflection using trigonometrical function by Rayleigh Ritz method



Step 1: Formulation of PE functions

Π = SE + WP

$$\Pi = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx - \int_0^L F\, y\, dx \qquad 01$$

Step 2: Selection of Displacement Model

$$y = C \sin\frac{\pi x}{L} \qquad 02$$

BC-1 x = 0 then y =0

BC-2 x=L then y =0

BC-3 x = L/2 then $\frac{\partial Y}{\partial x} = 0$

$$y = C \sin\frac{\pi x}{L} \qquad 03$$

$$\frac{\partial^2 y}{\partial x^2} = -C\,\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}$$

$$\left(\frac{\partial^2 y}{\partial x^2}\right)^2 = C^2\,\frac{\pi^4}{L^4}\sin\frac{\pi x}{L} \qquad 04$$

$$x = \frac{L}{2} \rightarrow\rightarrow\rightarrow y = y_m = C \qquad 05$$

$$\int_0^L p\, y\, dx = \int_0^L p\, C \sin\frac{\pi x}{L}\, dx = \frac{2FCL}{\pi}$$

Step 3: Substitute in PE

$$\Pi = \frac{EI}{2}\int_0^l C^2\,\frac{\pi^4}{L^4}\sin\frac{\pi x}{L} - \frac{2FCL}{\pi}$$

$$\Pi = \frac{C^2 EI\pi^4}{4L^3} - \frac{2FCL}{\pi} \qquad 06$$

Step 4: Applied Minimum Potential Energy

$$\frac{\partial \Pi}{\partial x} = \frac{2C\ \ EI\pi^4}{4L^3} - \frac{2FL}{\pi} = 0$$

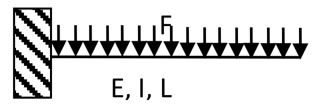
$$C = \frac{4FL^4}{EI\pi^5} \qquad 07$$

Substitute in Equation 03

$y = \frac{4FL^4}{EI\pi^5} \sin \frac{\pi x}{L}$ 08

$Y_m = \frac{4FL^4}{EI\pi^5} \quad = \frac{FL^4}{76.52\ EI}$

10. A cantilever beam of span L is subjected to a uniformly distributed load. Derive an equation for the deflection at free end by using Gelerkin technique. Assume polynomial function.



Step 1: Formulation of DE functions

$EI\frac{\partial^4 y}{\partial x^4} - F = 0$ 01

Step 2: Selection of Displacement Model

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$ 02

BC-1 x = 0, y=0 then a_0=0

BC-2 x = 0, $\frac{\partial Y}{\partial x} = 0$ then a_1=0

BC-3 x=L $\frac{\partial^2 Y}{\partial x^2} = 0$ $0 = 2a_2 + 6a_3 L + 12a_4 L^2$

$a_2 = -3a_3 L - 6a_4 L^2$ 03

BC-4 x=L $\frac{\partial^3 Y}{\partial x^3} = 0$ $0 = 6a_3 + 24\, a_4 L$

$a_3 = -4\, a_4 L$ 04

Substitute (4) into (3)

$a_2 = -3(-4\, a_4 L)\ L - 6a_4 L^2$

$a_2 = 12\, a_4 L^2 - 6a_4 L^2$

$a_2 = 6\, a_4 L^2$ 05

Substitute equation 4 and 5 in equation 1

$y = 6\, a_4 L^2 x^2 - 4\, a_4 L x^3 + a_4 x^4$

$y = a_4(x^4 - 4\, L x^3 + 6L^2 x^2)$ 06

$w_1 = x^4 - 4\, L x^3 + 6L^2 x^2$

$\frac{\partial y}{\partial x} = a_4(4x^3 - 12\, L x^2 + 12L^2 x\ )$

$\frac{\partial^2 y}{\partial x^2} = a_4(12x^2 - 24\, L x + 12L^2)$

$\frac{\partial^3 y}{\partial x^3} = a_4(24x - 24\, L)$

$\frac{\partial^4 y}{\partial x^4} = 24a_4$ 07

Sep 3: Substitute in DE and equating to R

$EI\ 24a_4 - F = R$ 08

Step 4: Applied Galerkin technique (weight residual technique)

$\int_0^l w_1 R\, dx = 0$ 09

$\int_0^l (x^4 - 4Lx^3 + 6L^2x^2)(EI\, 24a_4 - F)dx = 0$

$EI\, 24a_4 - F = 0$

$a_4 = \frac{F}{24EI}$ 10

Substitute in equation 6

$y = \frac{F}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$ 11

Maximum deflection is at x = L

$y = \frac{F}{24EI}(L^4 - 4LL^3 + 6L^2L^2)$

$y_m = \frac{F3L^4}{24EI}$

$y_m = \frac{FL^4}{8EI}$

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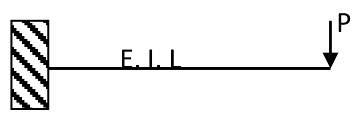
11. A cantilever beam of span L is subjected to a point load at free end. Derive an equation for the deflection at free end by using Galerkin method. Assume polynomial function



Step 1: Formulation of DE functions

$EI\frac{\partial^4 y}{\partial x^4} = 0$ 01

Step 2: Selection of Displacement Model

$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$ 02

BC-1 x = 0, y=0 then $a_0=0$

BC-2 x = 0, $\frac{\partial Y}{\partial x} = 0$ then $a_1=0$

BC-3 x=L $\frac{\partial^2 Y}{\partial x^2} = 0$ $0 = 2a_2 + 6a_3L + 12a_4L^2$

$a_2 = -3a_3L - 6a_4L^2$ 03

BC-4 x=L $\frac{\partial^3 Y}{\partial x^3} = \frac{-P}{EI}$ $\frac{-P}{EI} = 6a_3 + 24\,a_4L$

$a_3 = \frac{-P}{6EI} - 4\,a_4L$ 04

Substitute (4) into (3)

$a_2 = -3(\frac{-P}{6EI} - 4\,a_4L)\ L - 6a_4L^2$

$$a_2 = \frac{PL}{2EI} + 12\ a_4 L^2 - 6a_4 L^2$$

$$a_2 = \frac{PL}{2EI} + 6\ a_4 L^2 \qquad 05$$

Substitute equation 4 and 5 in equation 1

$$y = (\frac{PL}{2EI} + 6\ a_4 L^2)x^2 + \left(\frac{-P}{6EI} - 4\ a_4 L\right)x^3 + a_4 x^4$$

$$y = \frac{PL}{2EI}x^2 + 6\ a_4 L^2 x^2 - \frac{P}{6EI}x^3 - 4\ a_4 L x^3 + a_4 x^4$$

$$y = \frac{P}{2EI}(Lx^2 - \frac{x^3}{3}) + \ a_4(6L^2x^2 - 4\ Lx^3 + x^4) \qquad 06$$

$$w_1 = 6L^2x^2 - 4\ Lx^3 + x^4$$

$$\frac{\partial y}{\partial x} = \frac{P}{2EI}(2Lx \quad - \frac{3x^2}{3}) + \ a_4(12L^2x \quad - 12\ Lx^2 + 4x^3)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{P}{2EI}(2L - \frac{6x}{3}) + \ a_4(12L^2 - 24\ Lx \quad + 12x^2)$$

$$\frac{\partial^3 y}{\partial x^3} = \frac{P}{2EI}(0 - \frac{6}{3}) + \ a_4(24\ L + 24x \quad)$$

$$\frac{\partial^4 y}{\partial x^4} = \ 24a_4 \qquad 07$$

Sep 3: Substitute in DE and equating to R

EI $24a_4$ = R 08

Step 4: Applied Galerkin technique (weight residual technique)

$\int_0^l \mathrm{w}_1 \mathrm{R}\ \mathrm{dx} = 0$ 09

$\int_0^l (x^4 - \ 4\ Lx^3 + 6L^2x^2)24a_4 \mathrm{dx} = 0$

$a_4 = 0$ 10

Substitute in equation 6

$$y = \frac{P}{2EI}(Lx^2 - \frac{x^3}{3}) + \ 0(6L^2x^2 - 4\ Lx^3 + x^4)$$

$y = \frac{P}{2EI}(Lx^2 - \frac{x^3}{3})$

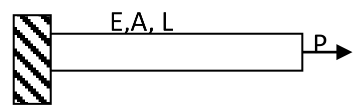
Maximum deflection is at x = L

$y_m = \frac{P}{2EI}(LL^2 - \frac{L^3}{3})$

$y_m = \frac{PL^3}{3EI}$

---

12. Use Galerkin method to find the displacement of the system shows in Fig.



Step 1: Formulation of DE functions

$EA\frac{\partial^2 u}{\partial x^2} = 0$ 01

Step 2: Selection of Displacement Model

$u = a_0 + a_1 x + a_2 x^2$ 02

BC-1 x = 0, y=0 then a_0=0

BC-2 x = L, $\frac{\partial Y}{\partial x} = \frac{P}{AE} = a_1 + 2a_2 L$

$a_1 = \frac{P}{AE} - 2a_2 L$

Substitute in 2

$u = \frac{Px}{AE} - a_2(x^2 - 2Lx)$

$u = \frac{Px}{AE} - a_2(x^2 - 2Lx)$ 03

$w_1 = x^2 - 2Lx$

$\frac{\partial u}{\partial x} = \frac{P}{AE} - a_2(2x \quad - 2L)$

$\frac{\partial^2 u}{\partial x^2} = 2a_2$

Sep 3: Substitute in DE and equating to R

$2EAa_2 = R$ 04

Step 4: Applied Galerkin technique (weight residual technique)

$\int_0^l w_1 R\, dx = 0$ 05

$\int_0^l (x^2 - 2Lx) \quad 2EAa_2 dx = 0$

$a_2 = 0$ 06

Substitute in equation 6

$u = \frac{Px}{AE} - 0(x^2 - 2Lx)$

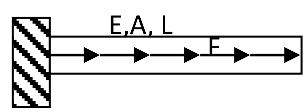
$u = \frac{Px}{AE}$

Maximum deflection is at x = L

$u_m = \frac{PL}{AE}$

---

13. Use Galerkin method to find the displacement of the system shows in Fig.



Step 1: Formulation of DE functions

$EA\frac{\partial^2 u}{\partial x^2} + F = 0$ 01

Step 2: Selection of Displacement Model

$u = a_0 + a_1 x + a_2 x^2$ 02

BC-1 x = 0, y=0 then a_0=0

BC-2 x = L, $\frac{\partial Y}{\partial x} = 0 = a_1 + 2a_2 L$

$a_1 = -2a_2 L$

Substitute in 2

$u = a_2(x^2 - 2Lx)$ 03

$w_1 = x^2 - 2Lx$

$\frac{\partial u}{\partial x} = a_2(2x \quad - 2L)$

$\frac{\partial^2 u}{\partial x^2} = 2a_2$

Sep 3: Substitute in DE and equating to R

$2EAa_2 + F = R$ 04

Step 4: Applied Galerkin technique (weight residual technique)

$\int_0^l w_1 R \, dx = 0$ 05

$\int_0^l (x^2 - 2Lx) \quad (2EAa_2 + F) \, dx = 0$

$(2EAa_2 + F) = 0$ 06

$a_2 = -\frac{F}{2EA}$

Substitute in equation 6

$u = -\frac{F}{2EA}(x^2 - 2Lx)$

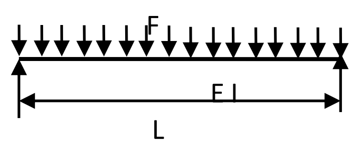
$u = \frac{F}{2EA}(2Lx - x^2)$

Maximum deflection is at x = L

$u_m = \frac{PL^2}{2AE}$

---

14. A simply supported beam is subjected UDL as shown in Figure. Determine the maximum deflection using Galerkin method.



Step 1: Formulation of DE functions

$EI \frac{\partial^4 y}{\partial x^4} - F = 0$ 01

Step 2: Selection of Displacement Model

$y = C \sin \frac{\pi x}{L}$ 02

$w_1 = \text{Sin} \frac{\pi x}{L}$

$\frac{\partial y}{\partial x} = C \frac{\pi}{L} \text{ Cos} \frac{\pi x}{L}$

$$\frac{\partial^2 y}{\partial x^2} = -C\frac{\pi^2}{L^2}\ \sin\frac{\pi x}{L}$$
$$\frac{\partial^3 y}{\partial x^3} = -C\frac{\pi^3}{L^3}\ \text{Cos}\frac{\pi x}{L}$$
$$\frac{\partial^4 y}{\partial x^4} = C\frac{\pi^4}{L^4}\ \text{Sin}\frac{\pi x}{L}$$

Sep 3: Substitute in DE and equating to R

$$\text{EI}\ C\frac{\pi^4}{L^4}\ \text{Sin}\frac{\pi x}{L} - \text{F} = \text{R}$$ 08

Step 4: Applied Galerkin technique (weight residual technique)

$$\int_0^l \text{Sin}\frac{\pi x}{L}(\text{EI}\ C\frac{\pi^4}{L^4}\ \text{Sin}\frac{\pi x}{L} - \text{F})\text{dx} = 0$$
$$\int_0^l(\text{EI}\ C\frac{\pi^4}{L^4}\ \text{Sin}^2\frac{\pi x}{L} - \text{F Sin}\frac{\pi x}{L})\text{dx} = 0$$
$$\int_0^l(\text{EI}\ C\frac{\pi^4}{L^4}\ (\frac{1-\cos\frac{2\pi x}{L}}{2}) - \text{F Sin}\frac{\pi x}{L})\text{dx} = 0$$

$$\text{EI}\ C\frac{\pi^4}{L^4}\ \frac{L}{2} + \text{F}\ \frac{\text{L}}{\pi}(-1-1) = 0$$

$$\text{EI}\ C\frac{\pi^4}{2L^3} - \frac{2\text{FL}}{\pi}\ = 0$$

$$C\ = \frac{4\text{FL}^4}{EI\pi^5}$$

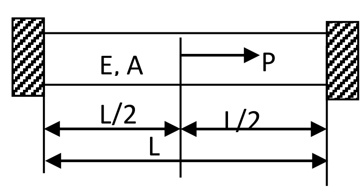
$$y = \frac{4\text{FL}^4}{EI\pi^5}\sin\frac{\pi x}{L}$$

Maximum deflection is at x = L/2

$$y = \frac{4\text{FL}^4}{EI\pi^5}\sin\frac{\pi}{2}$$
$$y_m = \frac{4\text{FL}^4}{EI\pi^5}$$

---

15. Figure shows a bar fixed at both ends subjected to an axial load. Determine the displacement at the loading point and the corresponding stress using Galerkin method. (E=1, A= 1, L=2 and p = 2)



$$\frac{d}{dx} EA \frac{\partial u}{\partial x} = 0 \qquad \frac{x=0 \text{ and } u=0}{x=L \text{ then } u=0}$$

Multiply by ϕ (virtual displacement) and integrate by parts

$$\int_0^L \frac{d}{dx} EA \frac{\partial u}{\partial x} dx\, \phi = 0$$

$$\left[EA \frac{\partial u}{\partial x}\, \phi\right]_0^L - \int_0^L EA \frac{\partial u}{\partial x}\frac{\partial \phi}{\partial x} dx = 0$$

$$P\phi_1 - \int_0^L EA \frac{\partial u}{\partial x}\frac{\partial \phi}{\partial x} dx = 0$$

Assume trail function

$u = a_0 + a_1 x + a_2 x^2$ 02

BC-1 x = 0, y=0 then a_0=0

BC-2 x = L, $y = 0 = a_1 + 2a_2 L = 0$

$a_1 = -2a_2 L$

$u = -2a_2 Lx + a_2 x^2$

$u = a_2(x^2 - Lx)$ = $u_1(x^2 - Lx)$

$\frac{\partial u}{\partial x} = u_1(2x - L)$ Similarly for ϕ 03

$\frac{\partial \phi}{\partial x} = \phi_1(2x - L)$ 04

$$P\phi_1 - \int_0^L EAu_1(2x - l)\phi_1(2x - l) \quad dx = 0$$

$$P\phi_1 - EAu_1 \int_0^L (4x^2 + L^2 - 4Lx)\phi_1 \quad dx = 0$$

$$P\phi_1 - EAu_1 \left[\frac{4x^3}{3} + xL^2 - \frac{4Lx^2}{2}\right]_0^L \phi_1 = 0$$

$$P\phi_1 - EAu_1 \left[\frac{4L^3}{3} + L^3 - \frac{L^3}{}\right] \quad \phi_1 = 0$$

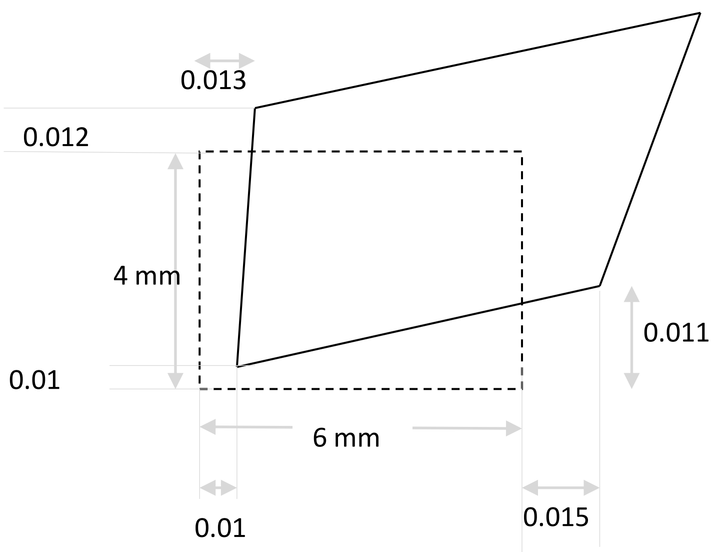
$$P\phi_1 - EAu_1 \left[\frac{L^3}{3}\right] \quad \phi_1 = 0$$

$\phi_1(P - EAu_1\left[\frac{L^3}{3}\right] = 0$

$u_1 = \frac{3P}{EAL^3} = \frac{3*2}{1*1*8} = \frac{3}{4}$

---

16. Estimate the three strains for given problem



$\varepsilon_x = \frac{du}{dx} = \frac{0.015-0.01}{6} = 0.000833$

$\varepsilon_y = \frac{dv}{dy} = \frac{0.012-0.01}{4} = 0.0005$

$\gamma_{xy} = \frac{dv}{dx} + \frac{du}{dy} = \frac{0.011-0.01}{6} + \frac{0.013-0.01}{4} = 0.000917$

17. Estimate the three strains and three stress for given problem. Assume given problem is plane stress method. (E=210 GPa, μ =0.3) (continuation with previous problem)

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{210e3}{1-0.3^2}\begin{bmatrix}1 & 0.3 & 0\\ 0.3 & 1 & 0\\ 0 & 0 & \frac{1-0.3}{2}\end{bmatrix}\begin{Bmatrix}0.0008333\\ 0.0005\\ 0.000917\end{Bmatrix} = \begin{bmatrix}226.9154\\ 173.0746\\ 74.06538\end{bmatrix} MPa$$

18. Solve x, y and z values using Gauss elimination method for the following equation

$$\begin{aligned} 2x &+ 4y - 2z = 2 \\ 4x &+ 9y - 3z = 8 \\ -2x &- 3y + 7z = 10 \end{aligned}$$

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ y + z &= 4 \quad R_2' \rightarrow R_2 - 2R_1 \\ y + 5z &= 12 \quad R_3' \rightarrow R_3 + R_1 \end{aligned}$$

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ y + z &= 4 \\ 4z &= 8 \quad R_3'' \rightarrow R_3' - R_2' \end{aligned}$$

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ y + z &= 4 \\ z &= 2 \end{aligned}$$

Divide by 2

$$\begin{aligned} 2x + 4y - 4 &= 2 \\ y + 2 &= 4 \\ z &= 2 \end{aligned}$$

Substitute Z values

$$\begin{aligned} 2x + 4y - 4 &= 2 \\ y &= 2 \\ z &= 2 \end{aligned}$$

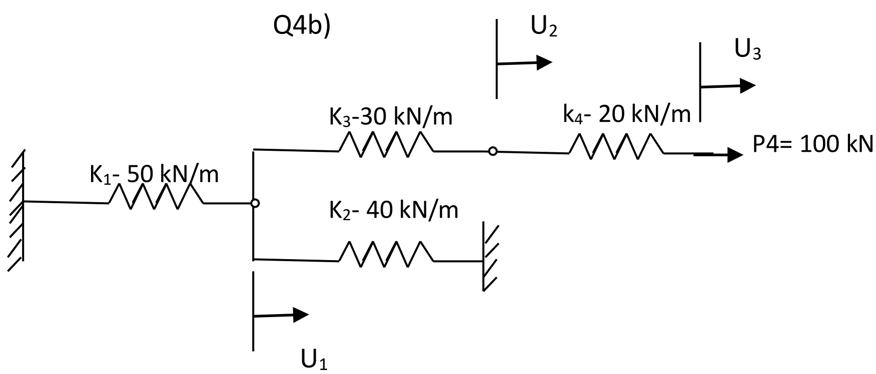
Y value obtained

$$\begin{aligned} 2x + 8 - 4 &= 2 \\ y &= 2 \\ z &= 2 \end{aligned}$$

Substitute y and z values

x = -1, y= 2 and z= 2

Q4b)



PE = SE + WP

$$SE = \frac{1}{2}k_1\delta_1^2 + \frac{1}{2}k_2\delta_2^2 + \frac{1}{2}k_3\delta_3^2 + \frac{1}{2}k_4\delta_4^2$$

$$SE = \frac{1}{2}k_1{u_1}^2 + \frac{1}{2}k_2u_1^2 + \frac{1}{2}k_3(u_2 - u_1)^2 + \frac{1}{2}k_4(u_3 - u_2)^2$$

$$WP = -P_4u_3$$

$$\pi = \frac{1}{2}k_1{u_1}^2 + \frac{1}{2}k_2u_1^2 + \frac{1}{2}k_3(u_2 - u_1)^2 + \frac{1}{2}k_4(u_3 - u_2)^2 - P_4u_3$$

$$\frac{\vartheta\pi}{\vartheta u_1} = k_1u_1 + k_2u_1 + k_3u_1 - k_3u_2 = 0$$

$$\frac{\vartheta\pi}{\vartheta u_1} = (k_1 + k_2 + k_3)u_1 - k_3u_2 = 0$$

$$(50 + 40 + 30)u_1 - 30u_2 = 0$$

$120u_1 - 30u_2 = 0$ ---------------------(1)

$$\frac{\vartheta\pi}{\vartheta u_2} = k_3u_2 - k_3u_1 + k_4u_2 - k_4u_3 = 0$$

$$-k_3u_1 + (k_3 + k_4)u_2 - k_4u_3 = 0$$

$$-30u_1 + (30 + 20)u_2 - 20u_3 = 0$$

$-30u_1 + 50u_2 - 20u_3 = 0$ ---------------------------(2)

$$\frac{\vartheta \pi}{\vartheta u_3} = -k_4 u_2 + k_4 u_3 - P_4 = 0$$

$$\frac{\vartheta \pi}{\vartheta u_3} = -20u_2 + 20u_3 = 100000$$ --------------------------(3)

From 1, 2 and 3 we get

u3=	9444.444
u2=	4444.444
u1=	1111.111