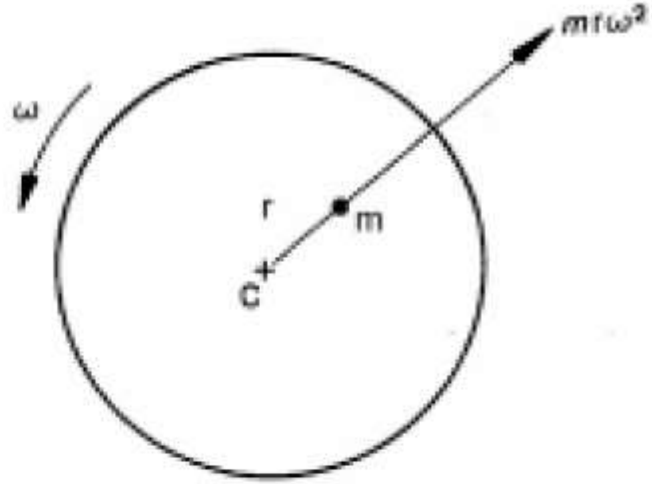
Need for Balancing

- In high speed machinery, the centrifugal forces are set up due to slightest eccentricity of rotors from the axis of rotation.
- These forces are considerable as the centrifugal force varies as the square of the angular velocity. ($F_c = m\omega^2 r$).
- The dynamic forces set up will produce noise and dangerous vibrations that destroy the bearings.
- Hence proper balancing is necessary for safe & smooth operation of machinery.



$$F = m*r*\omega^2 where \begin{cases} r = raduis & (m) \\ \omega = speed & (rad/\sec) \\ m = mass & (kg) \\ m*r = amount & (kg.m) \\ F = force & (N) \end{cases}$$

Static Balancing:

A system of rotating masses is said to be in static balance if the combined center of mass of lies on the axis of rotation.

Mathematically, a rotor is said to be in static balance if the algebraic sum of centrifugal forces is zero.

$i.e. \sum m\omega^2 r = 0$. As ω is same for all masses,

it can be written as $\sum mr = 0$

A system can be statically balanced with a single counter mass or balancing mass revolving in the same plane.

Dynamic Balancing:

A system of rotating masses is said to be in dynamic balance if there are no resultant unbalanced forces and couples acting on the body.

Mathematically, a rotor is said to be in dynamic balance if the algebraic sum of centrifugal forces is zero & also the algebraic sum of centrifugal couples is zero.

Mathematically, It can be written as;

$$\sum mr = 0 \cdots\cdots\cdots (i) \quad \text{(Force balance)}$$

and $$\sum mrl = 0 \cdots\cdots (ii) \quad \text{(Couple balance)}$$

For a system to be dynamically balanced , it requires at least two balancing masses revolving in different planes as two equations of equilibrium are to be satisfied.

Balancing of several masses revolving in same plane

A system of rotating masses is said to be in dynamic balance if there are no resultant unbalanced forces and couples acting on the body.

Mathematically, a rotor is said to be in dynamic balance if the algebraic sum of centrifugal forces is zero & also the algebraic sum of centrifugal couples is zero.

Mathematically, It can be written as;

$$\sum mr = 0 \cdots\cdots\cdots (i) \quad \text{(Force balance)}$$

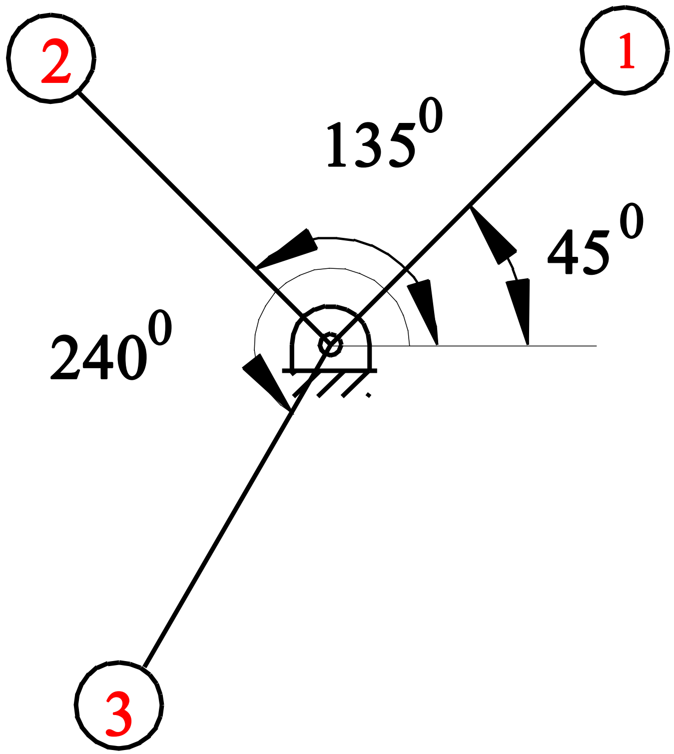
and $$\sum mrl = 0 \cdots\cdots (ii) \quad \text{(Couple balance)}$$

For a system to be dynamically balanced , it requires at least two balancing masses revolving in different planes as two equations of equilibrium are to be satisfied.

Problem 2

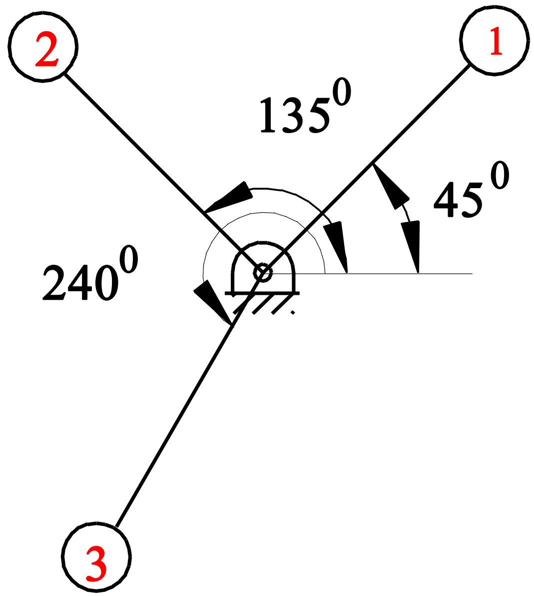
A circular disc mounted on a shaft carries three attached masses 4 kg, 3 kg and 2.5 kg at a radial distances 75 mm, 85 mm and 50 mm at the angular positions of 45^0, 135^0 and 240^0 respectively. The angular positions are measured counter-clockwise from the reference line along x-axis. Determine the amount of the counter mass at a radial distance of 75 mm required for static balance.

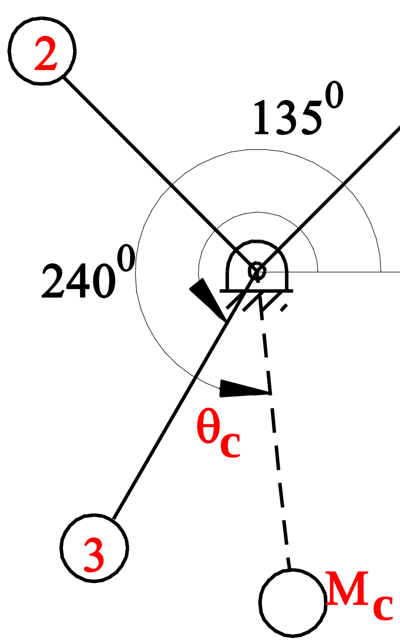
Space Diagram



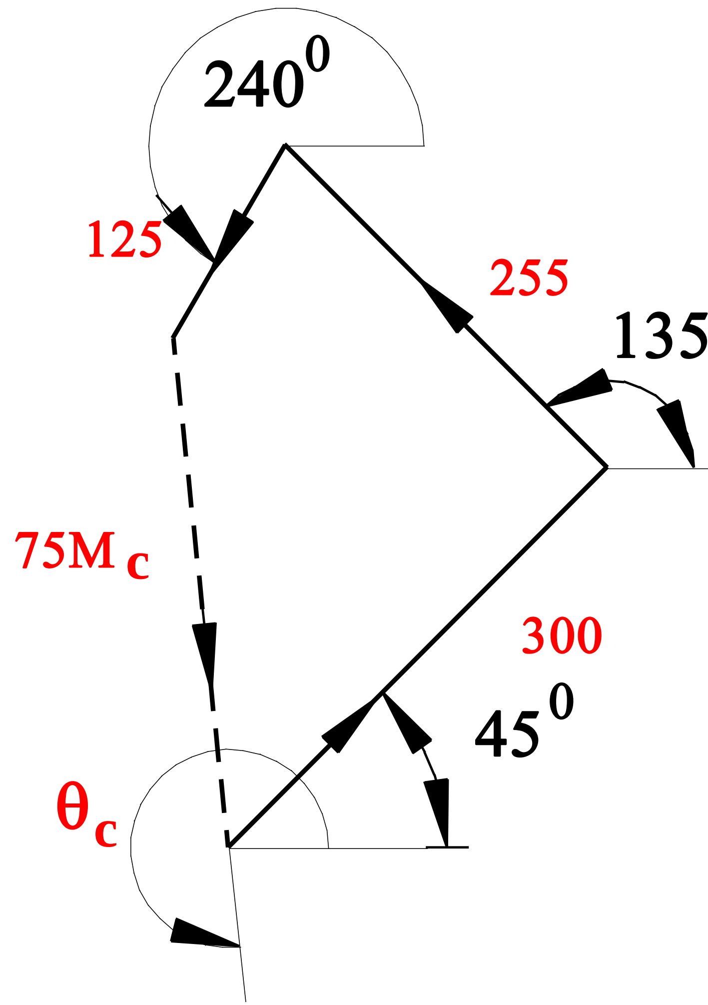
Force Table

Sl.No	Mass m (kg)	Radius (r) mm	Force m r (kg-mm)
1	4	75	300
2	3	85	255
3	2.5	50	125
4	M_c	75	$75M_c$





Force Polygon



Ans: M_c=3.81 Kg, θ_c =276^0

Analytical Solution :

For static balance, the summation of horizontal & Vertical components of forces must be individually equal to zero.

Sum of Horizontal components of forces = 0

$$\text{i.e.} \sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$$

$$\Rightarrow \left[(300 \times \cos 45^0) + (255\cos 135^0) + (125\cos 240^0) + (75\mathrm{M}_c \cos\theta_c) \right] = 0$$

$$\therefore \; M_c \cos\theta_c = 0.4091 \cdots\cdots\cdots (i)$$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$\Rightarrow \left[(300 \times \sin 45^0) + (255\sin 135^0) + (125\sin 240^0) + (75\mathrm{M}_c \sin\theta_c) \right] = 0$$

$$\therefore \; M_c \sin\theta_c = -3.789 \cdots\cdots\cdots (ii)$$

$\therefore\ M_c cos\theta_c = 0.4091 \cdots\cdots\cdots (i)$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$\Rightarrow \left[(300 \times \sin 45^0) + (255\sin 135^0) + (125\sin 240^0) + (75M_c \sin\theta_c)\right] = 0$$

$\therefore\ M_c sin\theta_c = -3.789 \cdots\cdots\cdots (ii)$

Squaring & adding (i) & (ii),

$$M_c = \sqrt{(0.4091)^2 + (-3.789)^2} = 3.81kg$$

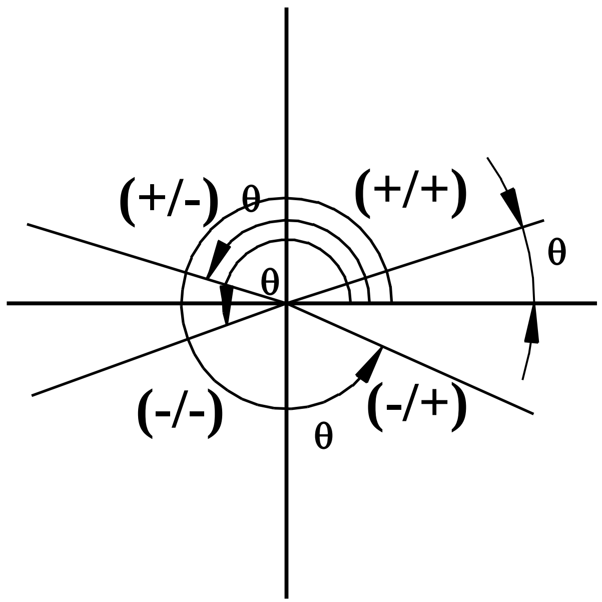
Also, *(ii) ÷ (i) gives*

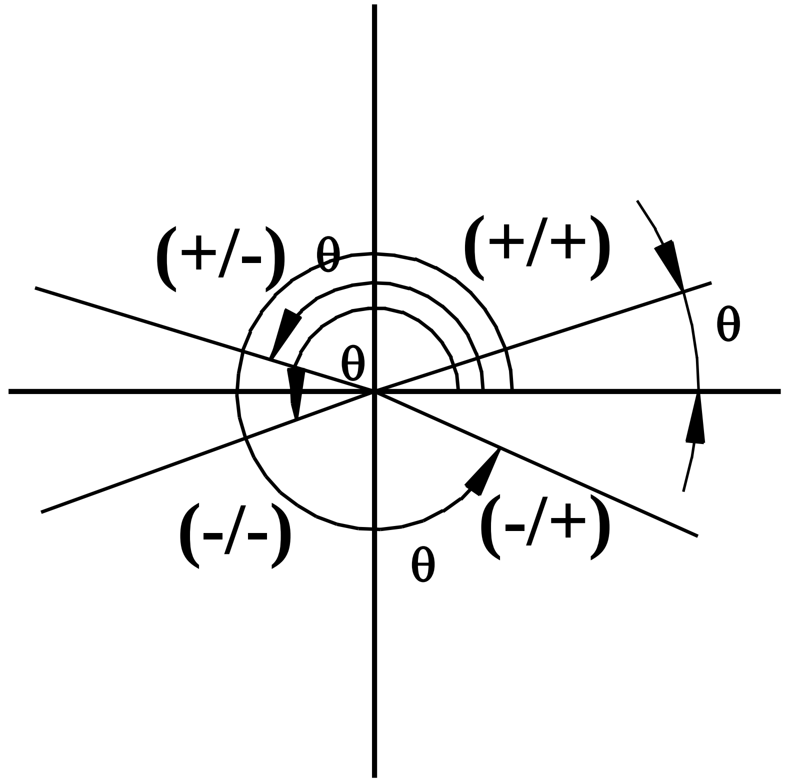
$$\frac{sin\ \theta_c}{cos\ \theta_c} = tan\ \theta_c = \frac{-3.789}{0.4091} = -9.2617$$

$$\therefore \theta_c = -83.84^0 = (360 - 83.84) = 276.16^0$$

(As numerator is - ve & denominator is + ve,

the angle lies in IV quadrant.)





Note :

(i) If both numerator & denominator are + ve, the angle is in I quadrant.

(ii) If numerator is + ve & denominator is - ve, the angle is in II quadrant.

(iii) If both numerator & denominator are - ve, the angle is in III quadrant.

(iv) If numerator is - ve & denominator is + ve, the angle is in IV quadrant.

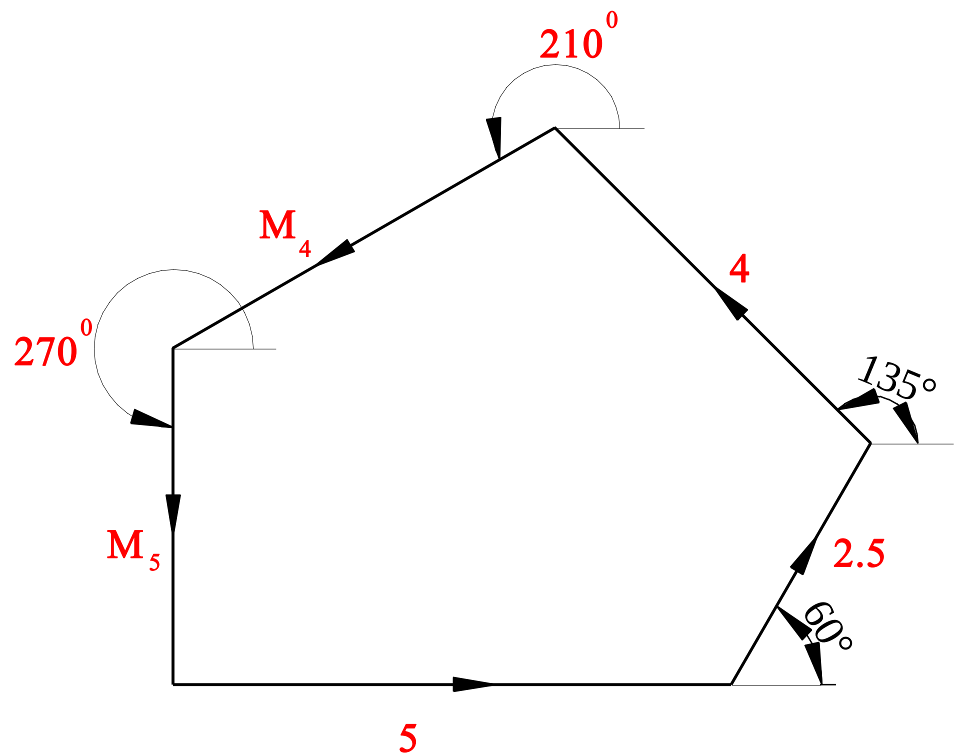
Problem 3

Five masses M_1, $M_{2,}$ $M_{3,}$ M_4 & M_5 revolve in same plane. Their magnitudes are 5 kg, 2.5 kg & 4 kg respectively and are attached ***same radial distances*** from the axis of rotation. Angular positions of $M_{2,}$ $M_{3,}$ M_4 & M_5 are 60^0, 135^0, 210^0, and 270^0 from M_1. Determine the values of M_4 & M_5 for static balance.

Graphical Solution

Force Table

Sl.No	Mass m (kg)	Radius (r) mm	Force m r (kg-mm)
1	5	1	5
2	2.5	1	2.5
3	4	1	4
4	M_4	1	M_4
5	M_5	1	M_5



Ans:

M_4=3.95 Kg,

M_5=3 Kg

Analytical Solution :

As the system of masses is balanced by itself, summation of horizontal & Vertical components of forces is individually equal to zero.

Sum of Horizontal components of forces = 0

i.e. $\sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$

$$\Rightarrow \left[(5\times\cos 0)+(2.5\cos 60^0)+(4\cos 135^0)+(\mathrm{M}_4\cos 210^0)+(\mathrm{M}_5\cos 270^0)\right]=0$$

$\therefore$ ***M_4 = 3.95 Kg***

Sum of vertical components of forces = 0

i.e. $\sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$

$$\Rightarrow \left[(5\times\sin 0)+(2.5\sin 60^0)+(4\sin 135^0)+(3.95\sin 210^0)+(\mathrm{M}_5\sin 270^0)\right]=0$$

$\therefore$ ***M_5 = 3.02 Kg***

Problem 4

Three masses of 8 kg, 12 kg & 15 kg attached at radial distances of 80 mm, 100mm and 60 mm respectively to a disc on a shaft are in static balance. Determine the angular positions of masses 12 kg & 15 kg relative to 8 kg mass.

Graphical Solution

Force Table

Sl.No	Mass m (kg)	Radius (r) mm	Force m r (kg-mm)
1	8	80	640
2	12	100	1200
3	15	60	900

Graphical Solution:

As the system of masses is in static balance, the force polygon is a triangle closed by itself.

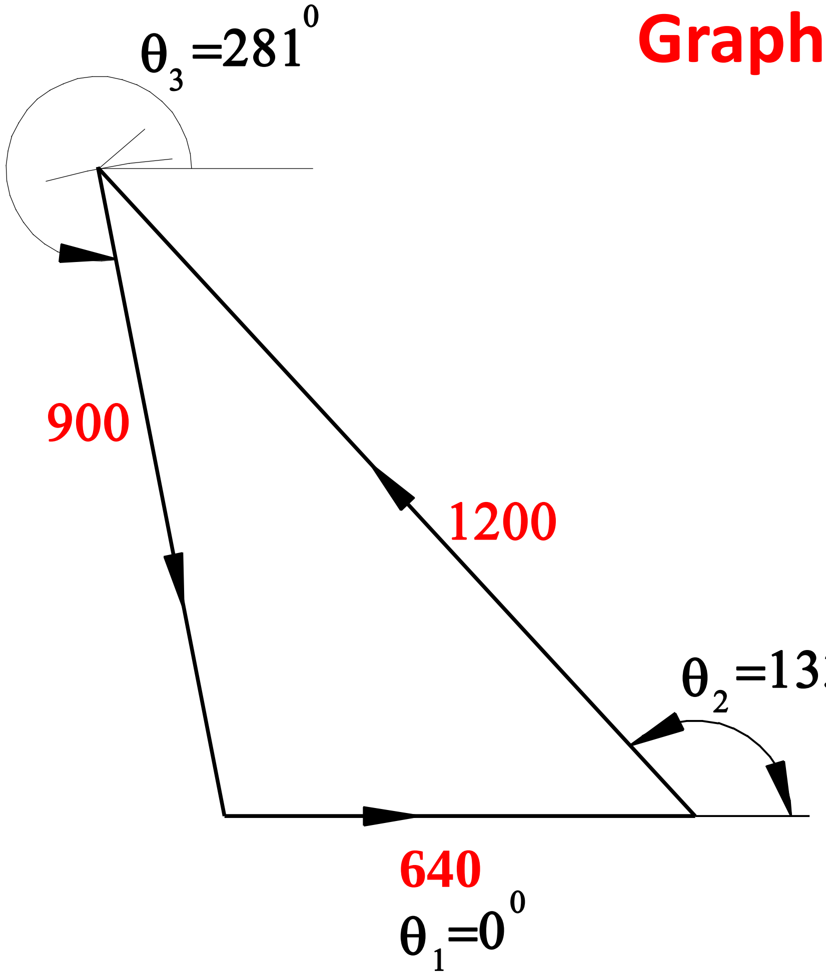
Let the angle of mass 1=θ_1 be 0^0.

Taking a suitable scale, draw vector m_1r_1 parallel to θ_1 (*i.e.* along the horizontal).

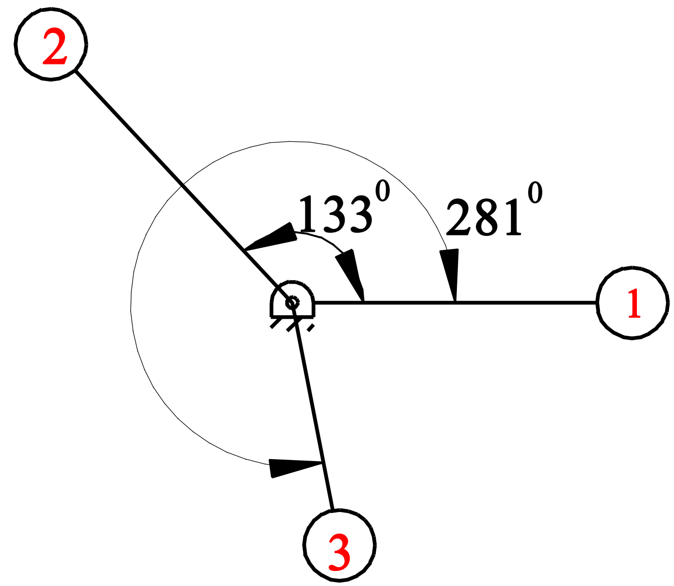
With radius equal to m_2r_2 & m_3r_3 draw two arcs to complete the traingle.

Then measure θ_2 and $\theta_{3.}$

Graphical Solution



Force Polygon



Relative angular positions of the masses

Analytical Solution :

As the system of masses is balanced by itself, summation of horizontal & Vertical components of forces is individually equal to zero.

Sum of Horizontal components of forces = 0

$$\text{i.e.} \sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$$

$$Hence\ \left[(640\times\cos 0)+(1200\cos\theta_2)+(900\cos\theta_3)\right]=0$$

$$\therefore 900\cos\theta_3 = -(640 + 1200\cos\theta_2)\ldots\ldots\ldots(i)$$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$Hence\ \left[(640\times\sin 0)+(1200\sin\theta_2)+(900\sin\theta_3)\right]=0$$

$$\therefore 900\sin\theta_3 = -1200\sin\theta_2\ldots\ldots\ldots(ii)$$

Squaring & adding (i) & (ii) ,

$$(900)^2 = (640)^2 + 2 \times 640 \times 1200 \cos \theta_2 + (1200)^2$$

$$\Rightarrow 810000 = 409600 + 1536000 \cos \theta_2 + 1440000$$

$$\therefore \cos \theta_2 = -0.6768 \Rightarrow \boldsymbol{\theta_2 = 132.6^0}$$

Substituting the value of θ_2 in the equations (i) & (ii),

$$\boldsymbol{900 \cos \theta_3 = -(640 + 1200 \cos 132.6)}$$

$$\therefore \boldsymbol{\cos \theta_3 = 0.1914.}$$

$$\boldsymbol{900 \sin\theta_3 = -1200 \sin 132.6}$$

$$\therefore \boldsymbol{\sin \theta_3 = -0.9815}$$

$$\Rightarrow \boldsymbol{\theta_3 = 281^0}$$

Balancing of several masses revolving in different planes (Dynamic Balancing)

- As the polygon law of forces is applicable to only coplanar forces, it is necessary to transfer all the forces to a reference plane (R.P) and then the force balance is achieved.
- But the transference of the forces to a reference plane leaves behind unbalanced couples, which also must be balanced by using graphical method or analytical method.
- Hence, for ***complete dynamic balance*** the force polygon & couple polygon must close. Analytically, the summation of horizontal & vertical components of centrifugal forces & couples must be zero.

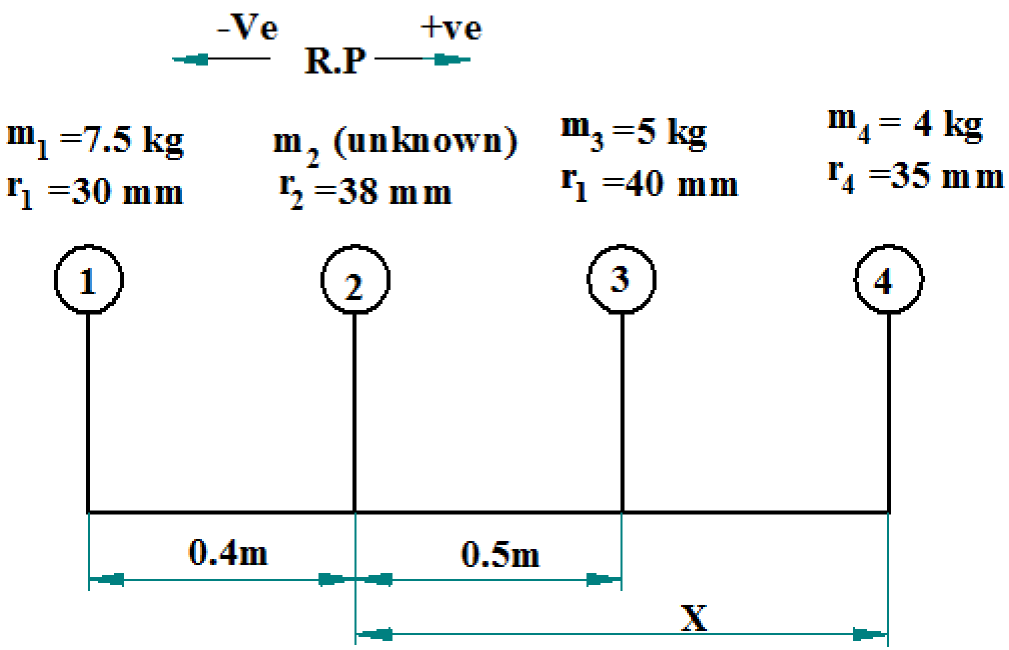
Problem 1

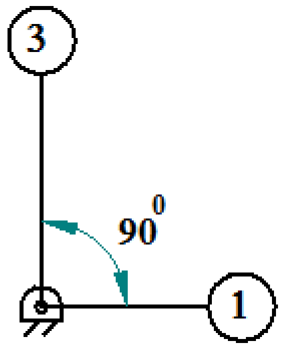
A rotating shaft carries four masses 1, 2, 3 & 4 which are radially attached to it. The mass centers are 30 mm, 38 mm, 40 mm and 35 mm respectively from the axis of rotation. The masses 1, 3 and 4 are 7.5, 5, & 4 kg respectively. The axial distance between the planes 1 and 2 is 400 mm & between 2 and 3 is 500 mm. The masses 1 & 3 are at right angles to each other. Find for complete balance,

(i) Angle between 1, 2 & 1, 4.

(ii) Axial distance between 3 & 4.

(iii) Magnitude of mass 2.

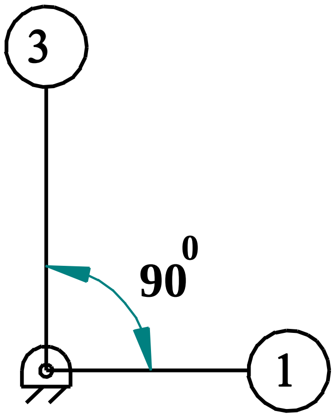


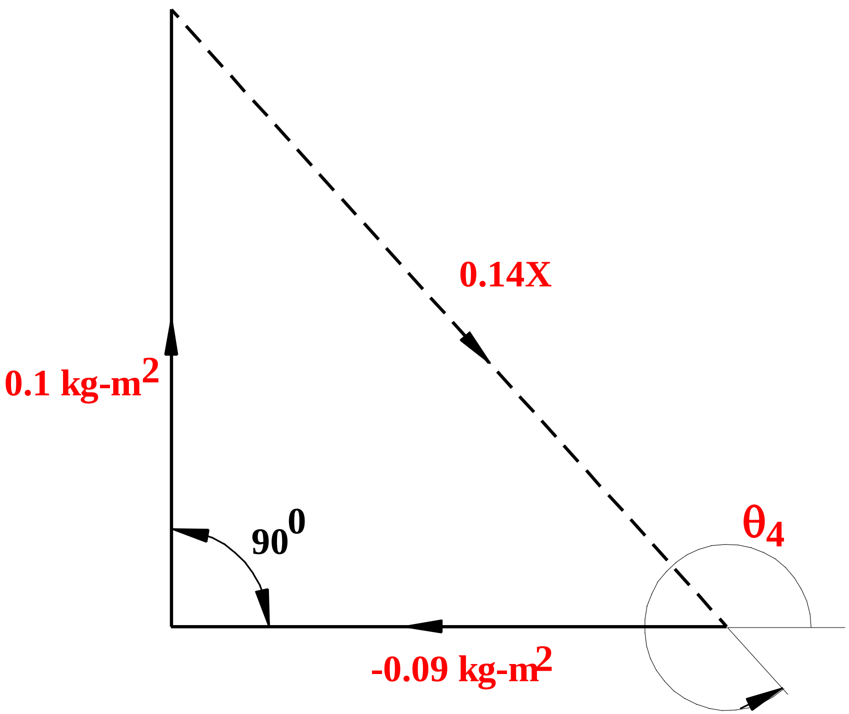


Here, $\theta_1=0^0$, θ_2 , θ_4 are unknown, $\theta_3=90^0$

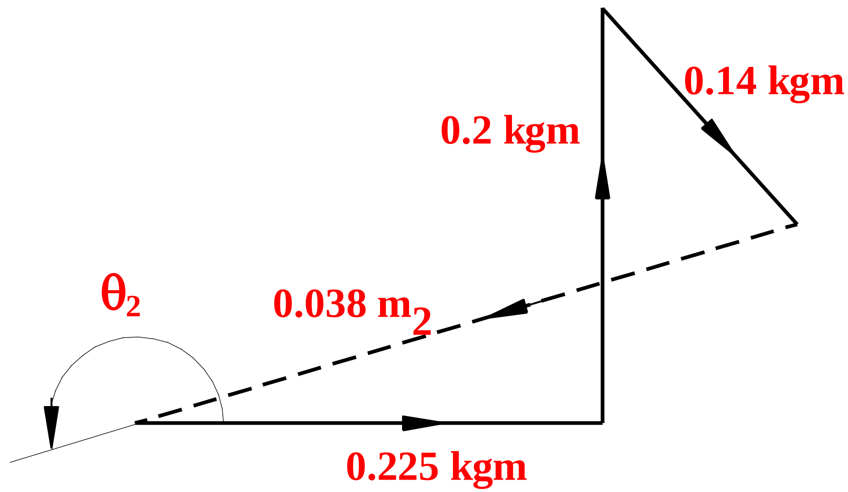
Plane	Mass m (kg)	Radius (r) mts	Force m r (kg-m)	Distance from R.P. (l) mts	Couple mrl kg-m^2
1	7.5	0.03	0.225	- 0.4	- 0.09
(* R.P) 2	m_2	0.038	$0.038m_2$	0	0
3	5	0.04	0.2	0.5	0.1
4	4	0.035	0.14	x	0.14x

If a couple is –ve , take the vector in the opposite direction of the mass





Couple Polygon



Force Polygon

From the couple polygon,
X=0.96 mts, θ_4=312^0

Hence distance between planes 3 & 4=0.96-0.5=0.46 mts

From the force polygon,
m_2 = 8.76 kg θ_2=197^0

Analytical Solution :

Note: As the couple table has only one unknown, it may be used to find the value of x & direction θ_4.

As the system of masses is balanced by itself, summation of horizontal & Vertical of components couples is individually equal to zero.

Sum of Horizontal components of couples = 0

i.e. $\sum C_H = 0 \Rightarrow \sum mrl\cos\theta = 0$

$$Hence\ \left[(-0.09\times\cos 0) + 0 + (0.1\cos 90^0) + (0.14x\cos\theta_4)\right] = 0$$

$\therefore$ ***0.14x cosθ_4 = 0.09 ....(i)***

Sum of vertical components of couples = 0

i.e. $\sum C_V = 0 \Rightarrow \sum mrl\sin\theta = 0$

$$Hence\ \left[(-0.09\times\sin 0) + 0 + (0.1\sin 90^0) + (0.14x\sin\theta_4)\right] = 0$$

$\therefore$ ***0.14x sinθ_4 = -0.1 ....(ii)***

Squaring & adding (i) & (ii),

$$0.14x\sqrt{\cos^2\theta_4+\sin^2\theta_4}=\sqrt{(-0.09)^2+(0.1)^2}$$

$\Rightarrow 0.14x=0.13454.$ Hence x=0.961 mts.

$\therefore$ Distance between planes of 3 & 4=0.961-0.5=***0.461 meters***.

Angular Position of mass 4 (θ_4)

Dividing eqn (ii) by (i), we get,

$$\theta_4=\tan^{-1}\left(\frac{-0.1}{0.09}\right)=-48^0.$$

As numerator is -ve & denominator is +ve, the angle is in the Fourth quadrant.

Hence $\theta_4=(360-48)=312^0$ measured ccw w.r.t. 1.

To find the value of Mass m_2 & direction θ_2

As the system of masses is balanced by itself, summation of horizontal & Vertical of components forces is individually equal to zero.

Sum of Horizontal components of forces = 0

$$\text{i.e.} \sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$$

$$Hence\ \left[(0.225\times\cos 0) + 0.038m_2\cos\theta_2 + (0.2\cos 90^0) + (0.14\times 0.961\cos 312^0)\right] = 0$$

$$\therefore \boldsymbol{m_2\cos\theta_2 = -8.29} \ldots\ldots(iii)$$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$Hence\ \left[(0.225\times\sin 0) + 0.038m_2\sin\theta_2 + (0.2\sin 90^0) + (0.14\times 0.961\sin 312^0)\right] = 0$$

$$\therefore \boldsymbol{m_2\sin\theta_2 = -2.632} \ldots\ldots(iv)$$

Squaring & adding (iii) & (iv),

$$\boldsymbol{m}_2\sqrt{\cos^2\theta_2 + \sin^2\theta_2} = \sqrt{(-8.29)^2 + (-2.632)^2}$$

$$\Rightarrow \boldsymbol{m_2 = 8.7kg.}$$

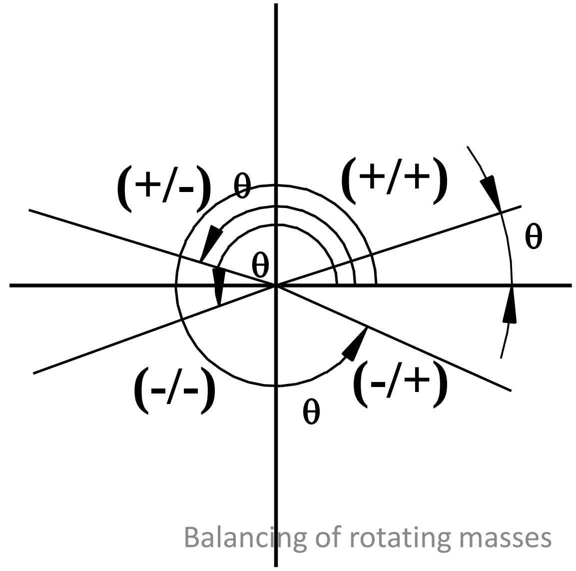
Angular Position of mass 2(θ_2)

Dividing eqn (ii) by (i), we get, (taking absolute values)

$$\theta_2 = \tan^{-1}\left(\frac{-2.632}{-8.29}\right) = 17.61^0$$

As both numerator & denominator is -ve, the angle is in the third quadrant.

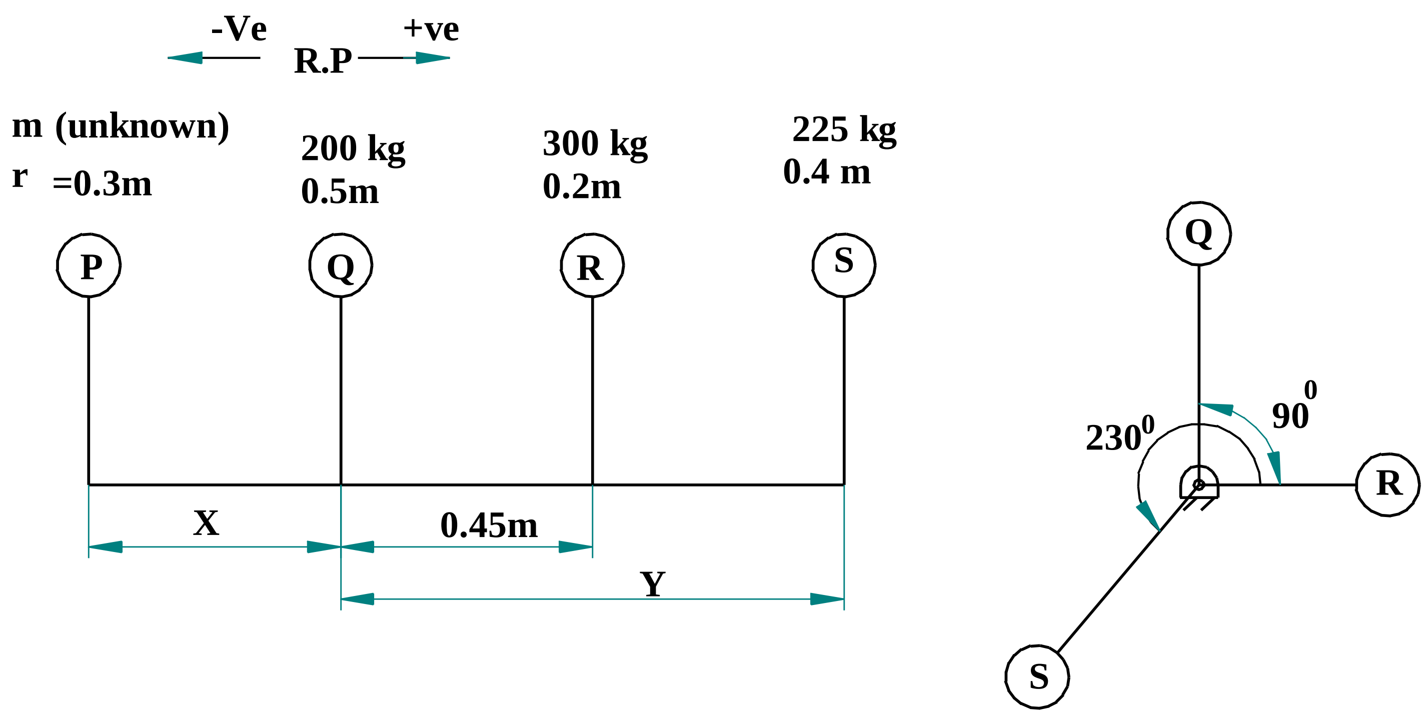
Hence $\theta_2 = (180 + 17.61) = 197.61^0$ measured ccw w.r.t. 1.



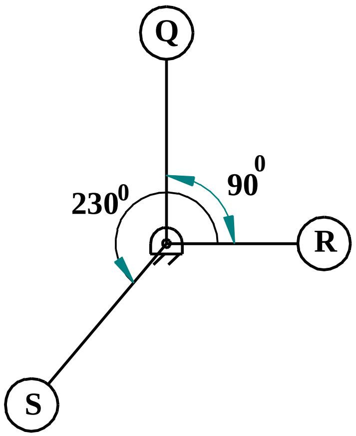
Problem 2

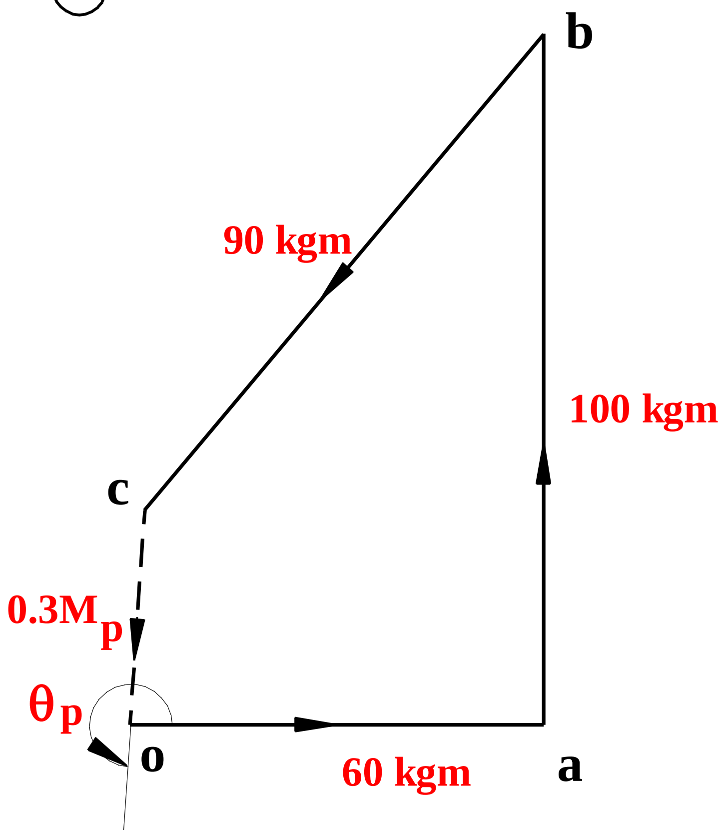
P, Q, R & S are the four masses rotating in different planes arranged to give complete balance. Planes containing Q & R are 450 mm apart. The masses Q & S make angles of 90^0 and 230^0 respectively w.r.t R in the same sense. Find where the planes containing P & S must be placed and also the magnitude and angular position of mass P.

Plane	**Mass (kg)**	**Radius (m)**
P	**M_p**	**0.3**
Q	**200**	**0.5**
R	**300**	**0.2**
S	**225**	**0.4**

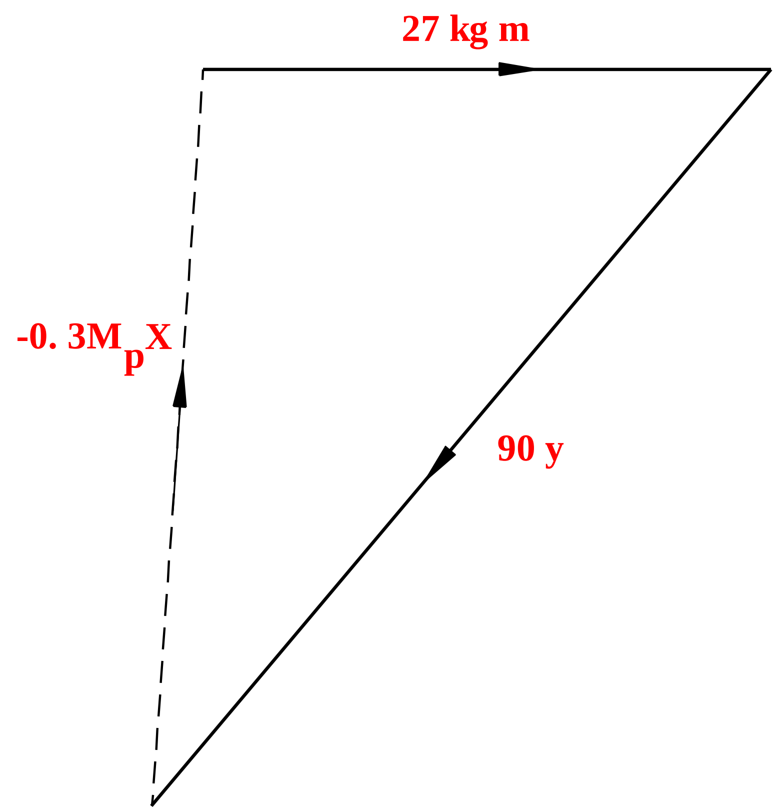

-Ve
+ve
R.P
m (unknown)
r =0.3m
200 kg
0.5m
300 kg
0.2m
225 kg
0.4 m
P
Q
R
S
X
0.45m
Y
Q
90⁰
230⁰
R
S

Plane	Mass m (kg)	Radius (r) mts	Force m r (kg-m)	Distance from R.P. (l) mts	Couple mrl kg-m^2
P	M_p	0.3	$0.3M_p$	- x	$-0.3M_px$
(R.P)*Q	200	0.5	100	0	0
R	300	0.2	60	0.45	27
S	225	0.4	90	y	90y





Force Polygon



Couple Polygon

From the force polygon,
M_P = 103.76 kg θ_p =266⁰

From the couple polygon,
X= 1.13 meters, Y= 0.51 meters

Analytical Solution:

Note:As the Force table has only one unknown, it may be used to find the value of mass M_p & direction θ_p.

As the system of masses is balanced by itself, summation of horizontal & Vertical of components force is individually equal to zero.

Sum of Horizontal components of forces = 0

$$\text{i.e.} \sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$$

$$\textit{Hence}\ \left[(0.3M_p\cos\theta_p) + 100\cos 90^0 + (60\cos 0^0) + (90\cos 230^0)\right] = 0$$

$$\therefore M_p\cos\theta_p = -7.164 \ldots\ldots(i)$$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$\textit{Hence}\ \left[(0.3M_p\sin\theta_p) + 100\sin 90^0 + (60\sin 0^0) + (90\sin 230^0)\right] = 0$$

$$\therefore M_p\sin\theta_p = -103.52 \ldots\ldots(ii)$$

Squaring & adding (i) & (ii),

$$M_p\sqrt{\cos^2\theta_p + \sin^2\theta_p} = \sqrt{(-7.164)^2 + (-103.52)^2}$$

$$\Rightarrow M_p = 103.76kg$$

Angular Position of mass P(θ_p)

Dividing eqn (ii) by (i), we get, (taking absolute values)

$$\theta_p = \tan^{-1}\left(\frac{-103.52}{-7.164}\right) = 86^0$$

As both numerator & denominator are -ve, angle is in the Third quadrant.

Hence $\theta_p = 180^0 + 86 = 266^0$ measured ccw w.r.t. 1.

Note :

(i) If both numerator & denominator are + ve, the angle is in I quadrant.

(ii) If numerator is + ve & denominator is - ve, the angle is in II quadrant.

(iii) If both numerator & denominator are - ve, the angle is in III quadrant.

(iv) If numerator is - ve & denominator is + ve, the angle is in IV quadrant.

As the system of masses is balanced by itself, summation of horizontal & Vertical of components couples is individually equal to zero.

Sum of Horizontal components of couples = 0

i.e. $\sum C_H = 0 \Rightarrow \sum mrl\cos\theta = 0$

$$Hence\ \left[(-0.3\times103.76\boldsymbol{x}\cos 266^0)+0+(27\cos 0^0)+(90\boldsymbol{y}\ \cos 230^0)\right]=0$$

$2.17x-57.85y=-27 \Rightarrow$ ***x - 26.66y = -12.44 ……(iii)***

Sum of vertical components of couples = 0

i.e. $\sum C_V = 0 \Rightarrow \sum mrl\sin\theta = 0$

$$Hence\ = \left[(-0.3\times103.76\boldsymbol{x}\sin 266^0)+0+(27\sin 0^0)+(90\boldsymbol{y}\ \sin 230^0)\right]=0$$

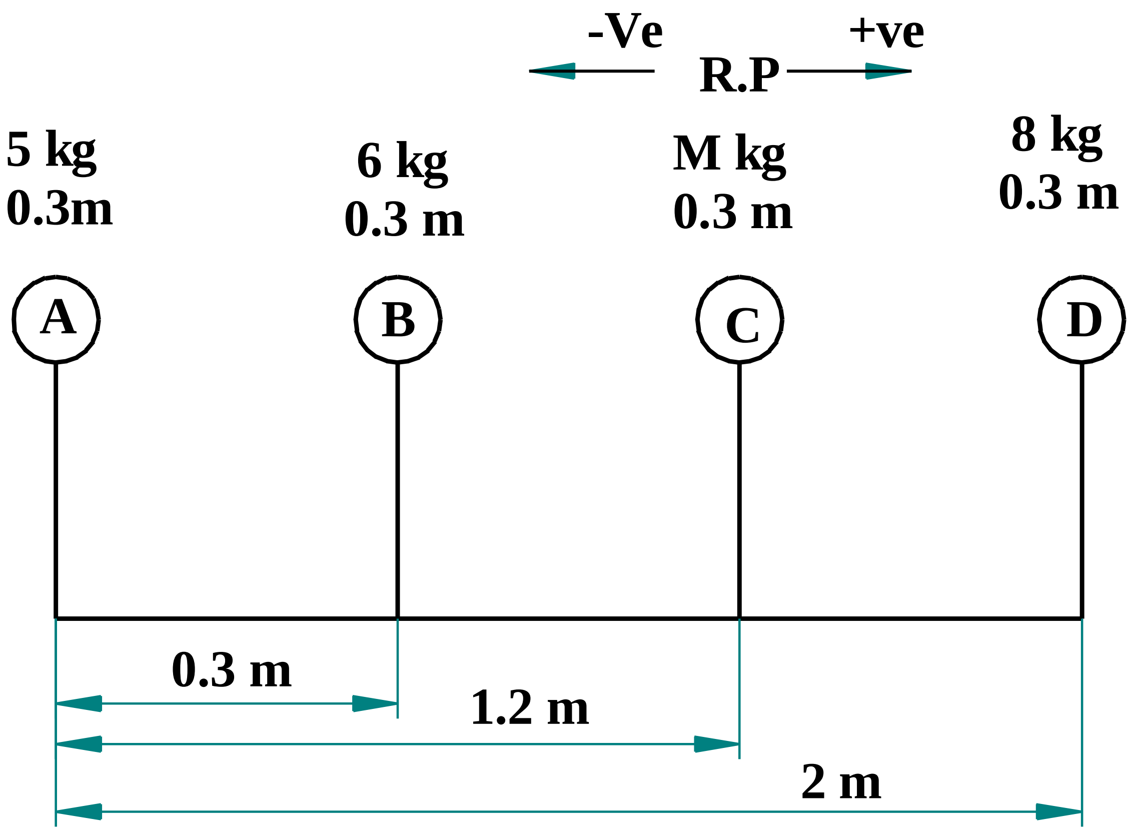
$31.05x-68.944y=0 \Rightarrow$ ***x - 2.22y = 0 ……(iv)***

solving equations(iii) & (iv) ,

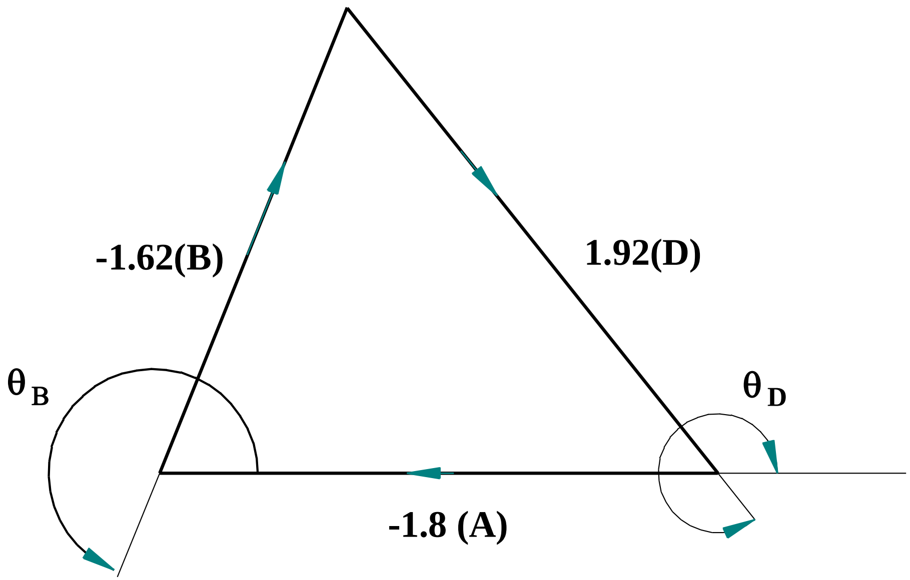
we get y = 0.51mts, x = 1.13 mts

Problem 3

A system of rotating masses which is in complete dynamic balance has magnitudes of **5, 6 M** & **8** kg and revolve in planes A, B, C & D. The planes B, C and D are at distances of 0.3 m, 1.2 m and 2.0 m respectively from A. All the masses are at the same radii of 0.3 m. Find the magnitudes of M and relative angular position of all the masses for complete balance.


-Ve
+ve
R.P
5 kg
0.3m
6 kg
0.3 m
M kg
0.3 m
8 kg
0.3 m
A
B
C
D
0.3 m
1.2 m
2 m

Plane	Mass m (kg)	Radius (r) mts	Force m r (kg-m)	Distance from R.P. (l) mts	Couple mrl $kg\text{-}m^2$
A	5	0.3	1.5	- 1.2	-1.8
B	6	0.3	1.8	-0.9	-1.62
(R.P) *C	M	0.3	0.3M	0	0
D	8	0.3	2.4	0.8	1.92

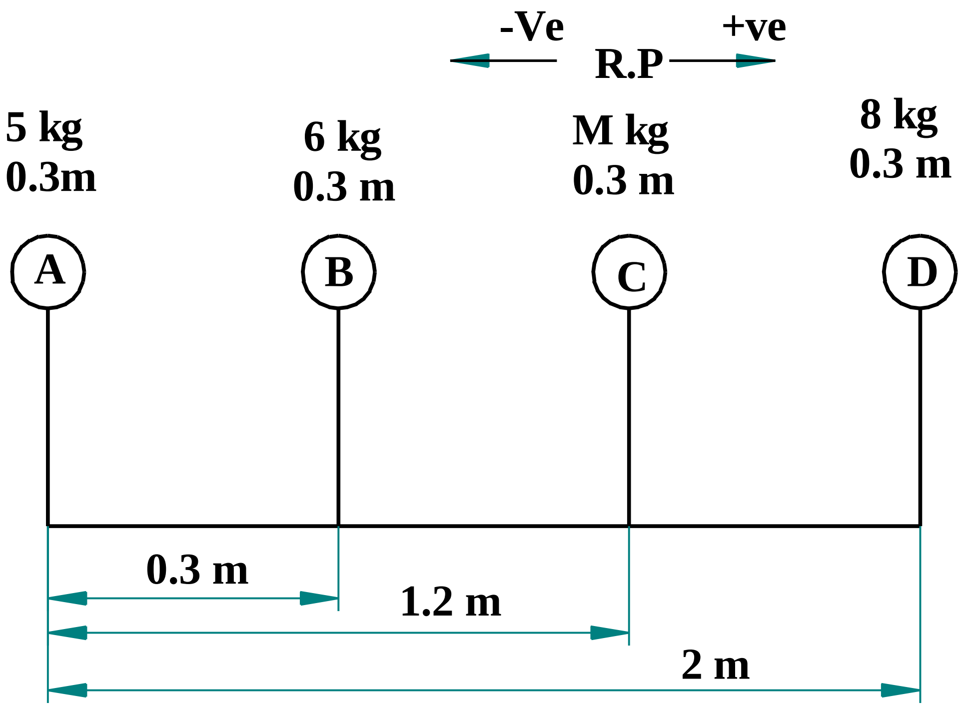


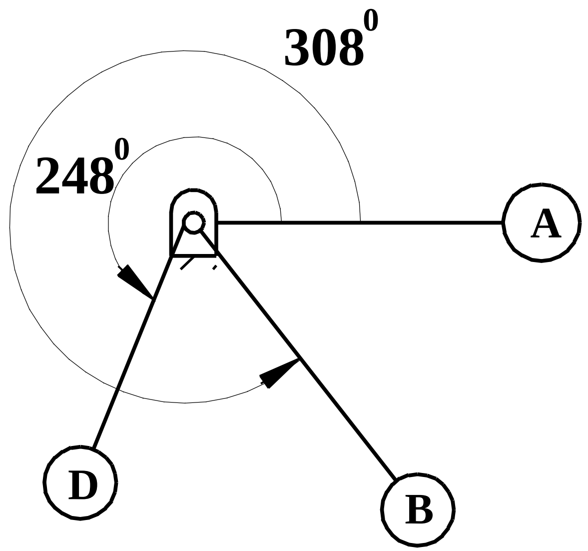
COUPLE POLYGON

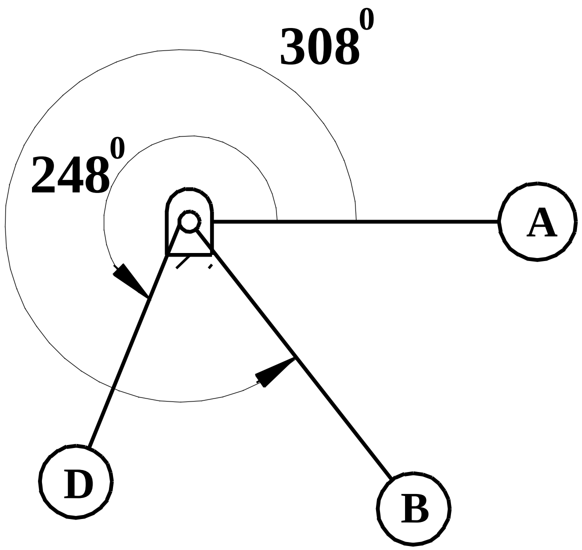
Let the angle of mass A=0 deg. i.e from left to right.
But as the couple is negative, take the vector in the opposite direction.
Again the vector B represents -ve couple.
Hence the angle of mass B from reference mass (A) should be measured opposite to the direction of vector.

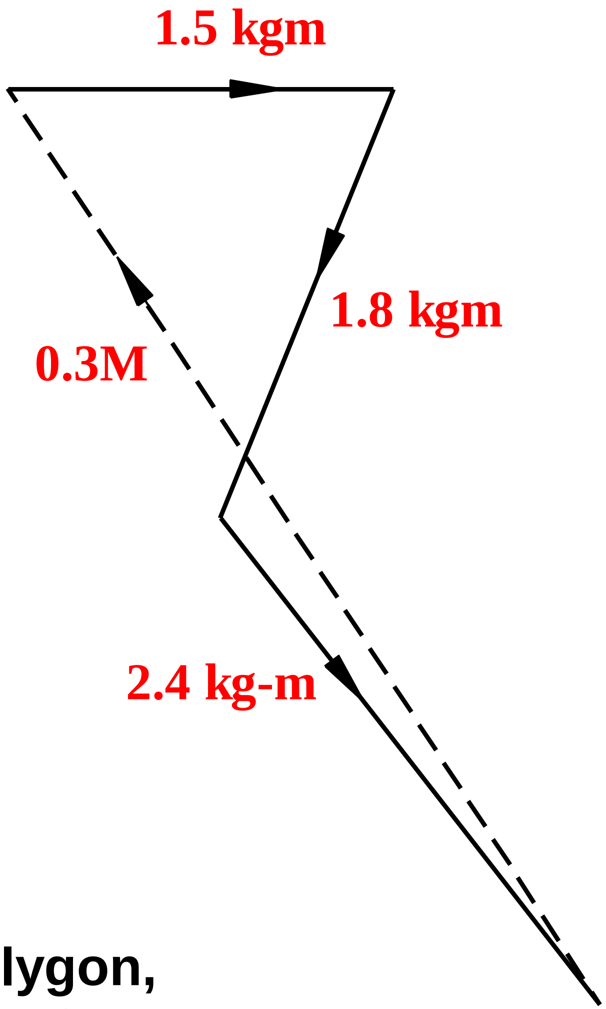
As the system of masses is completely balanced, the couple polygon must be a closed polygon.
From the couple polyon,
we get angle of mass D=308 deg,
& angle of mass B=248 deg measured from A in ccw direction.

Hence $\theta_A = 0$ deg $\theta_B = 248$ deg $\theta_D = 308$ deg


-Ve
+ve
R.P
5 kg
0.3m
6 kg
0.3 m
M kg
0.3 m
8 kg
0.3 m
A
B
C
D
0.3 m
1.2 m
2 m


308°
248°
A
D
B





From the force polygon,
M = 14.13 kg θ_M =123^0

Force Polygon

Analytical Solution :

As the system of masses is balanced by itself, summation of horizontal & Vertical components of couples is individually equal to zero.

Sum of Horizontal components of couples = 0

$$\text{i.e.} \sum C_H = 0 \Rightarrow \sum mrl\cos\theta = 0$$

$$Hence\ \left[(-1.8\times\cos 0)+(-1.62\cos\theta_B)+(1.92\cos\theta_D)\right]=0$$

$$\therefore \mathbf{1.92\ cos\ \theta_D = (1.8 + 1.62\ cos\ \theta_B) \ldots\ldots\ldots (i)}$$

Sum of vertical components of couples = 0

$$\text{i.e.} \sum C_V = 0 \Rightarrow \sum mrl\sin\theta = 0$$

$$Hence\ \left[(-1.8\times\sin 0)+(-1.62\sin\theta_B)+(1.92\sin\theta_D)\right]=0$$

$$\therefore \mathbf{1.92\ sin\ \theta_D = 1.62\ sin\ \theta_B \ldots\ldots\ldots (ii)}$$

$$1.92 \cos \theta_D = (1.8 + 1.62 \cos \theta_B) \ldots\ldots\ldots (i)$$

$$1.92 \sin \theta_D = 1.62 \sin \theta_B \ldots\ldots\ldots (ii)$$

Squaring & adding (i) & (ii),

$$(1.92)^2 = (1.8)^2 + 2 \times 1.8 \times 1.62 \cos \theta_B + (1.62)^2$$

$$\Rightarrow 3.6864 = 3.24 + 5.832 \cos \theta_B + 2.6244$$

$$\therefore \cos \theta_B = -0.3734 \Rightarrow \theta_B = 111.93^0 \approx 112^0 \text{ OR } \theta_B = 248^0$$

Substituting the value of θ_B in the equations (i) & (ii),

$$1.92 \cos \theta_D = (1.8 + 1.62 \cos 112^0)$$

$$\therefore \cos \theta_D = 0.6214.$$

$$1.92 \sin\theta_D = 1.62 \sin 112^0 \quad (\text{Or } 1.62 \sin 248^0)$$

$$\therefore \sin \theta_D = 0.7823 \qquad (\text{Or } -0.7823)$$

$$\Rightarrow \theta_D = 51.57^0 \quad \text{Or} \quad 308.43^0$$

Select any one pair of values and proceed to force balance

i.e $\theta_B = 248^0$ & $\theta_D = 308.43^0$ (OR)

$\theta_B = 112^0$ & $\theta_D = 51.57^0$

To find the value of Mass M & direction θ_M

' Selecting $\theta_B = 248^0$ & $\theta_D = 308^0$)

As the system of masses is balanced by itself, summation of horizontal & Vertical of components forces is individually equal to zero.

Sum of Horizontal components of forces = 0

$$\text{i.e.} \sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$$

$$Hence \left[(1.5\times\cos 0)+1.8\cos 248^0+(0.3M\cos\theta_M)+(2.4\times\cos 308^0)\right]=0$$

$$\therefore M\cos\theta_M = -7.68 \ldots\ldots(i)$$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$Hence \left[(1.5\times\sin 0)+1.8\sin 248^0+(0.3M\sin\theta_M)+(2.4\times\sin 308^0)\right]=0$$

$$\therefore M\sin\theta_M = 11.87 \ldots\ldots(ii)$$

Squaring & adding (iii) & (iv) ,

$$M\sqrt{\cos^2\theta_M+\sin^2\theta_M} = \sqrt{(-7.68)^2+(11.87)^2}$$

$$\Rightarrow m_2 = 14.13kg.$$

Angular Position of mass M(θ_M)

Dividing eqn (ii) by (i), we get,

$$\theta_M = \tan^{-1}\left(\frac{11.87}{-7.68}\right) = -57.1^0 = (180 - 57.1) = 122.9^0 \approx 123^0$$

(As the numerator is +ve & denominator is -ve, angle is in the second quadrant.)

(Hence $\theta_M = (180^0 - 57.1^0) = 123^0$ measured ccw w.r.t. 1.)

Note :

(i) If both numerator & denominator are + ve, the angle is in I quadrant.

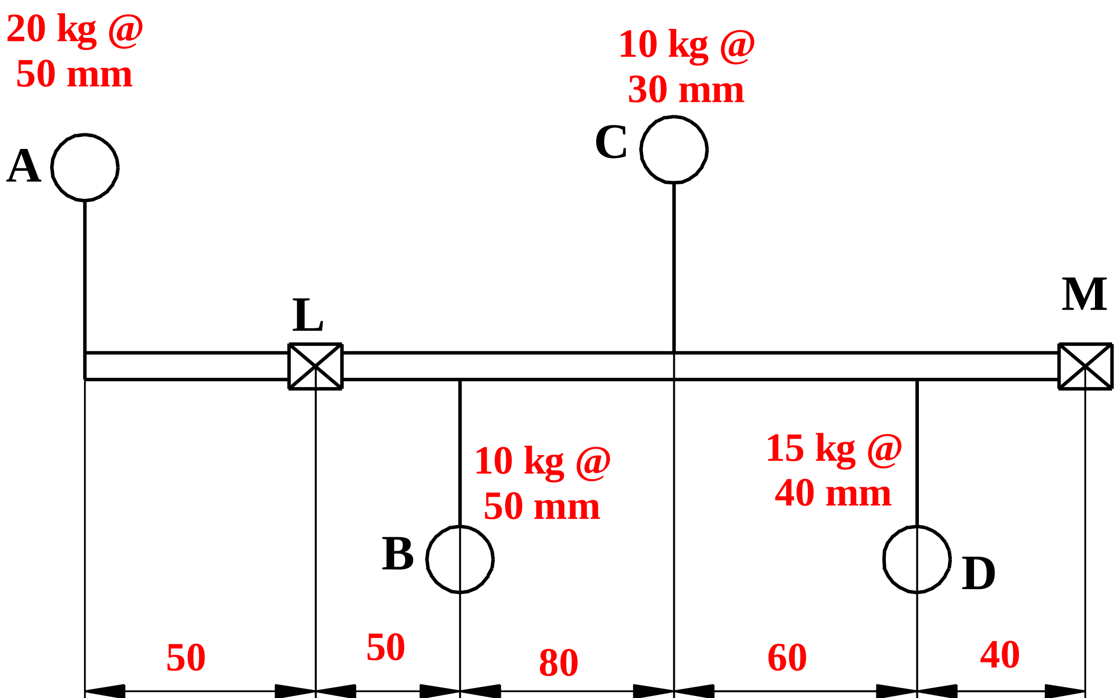
(ii) If numerator is + ve & denominator is - ve, the angle is in II quadrant.

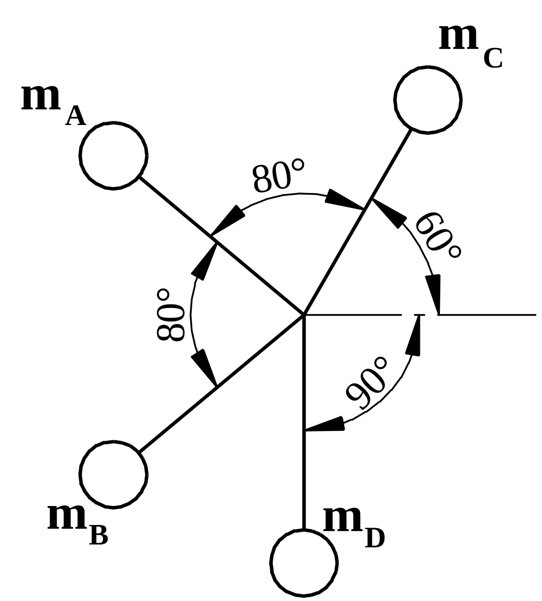
(iii) If both numerator & denominator are - ve, the angle is in III quadrant.

(iv) If numerator is - ve & denominator is + ve, the angle is in IV quadrant.

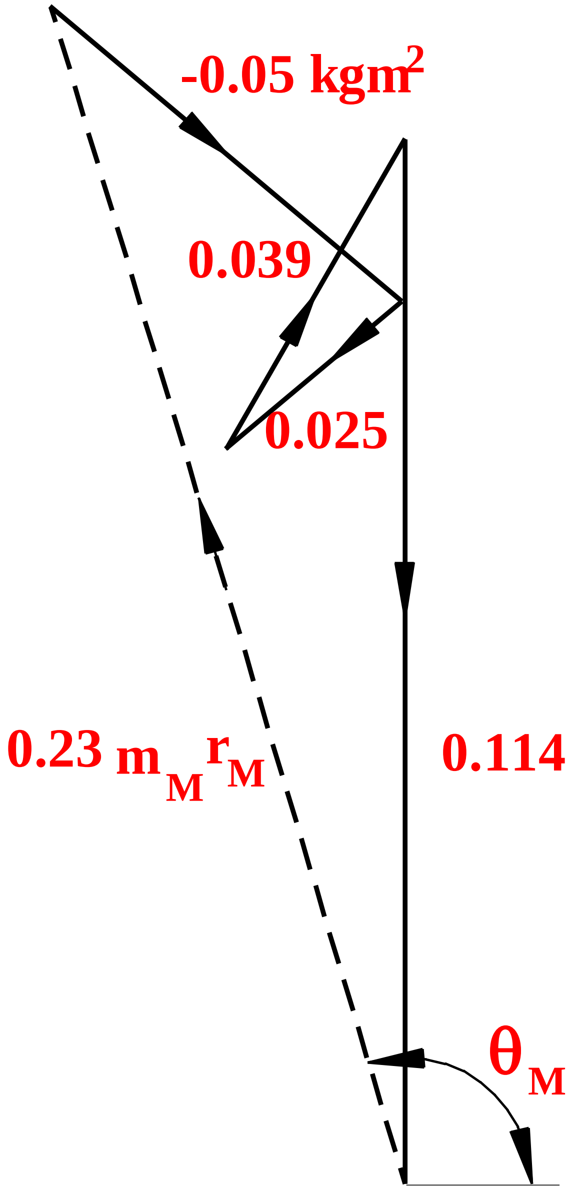
Problem 4

The fig shows a system of four unbalanced masses. Determine the dynamic force (reaction) at the bearings if the rotor speed is 600 rpm. Take the masses in planes A, B, C & D as 20 kg, 10 kg, 10 kg & 15 kg respectively. Their radii of rotation are 50 mm, 50 mm, 30 mm & 40 mm respectively.

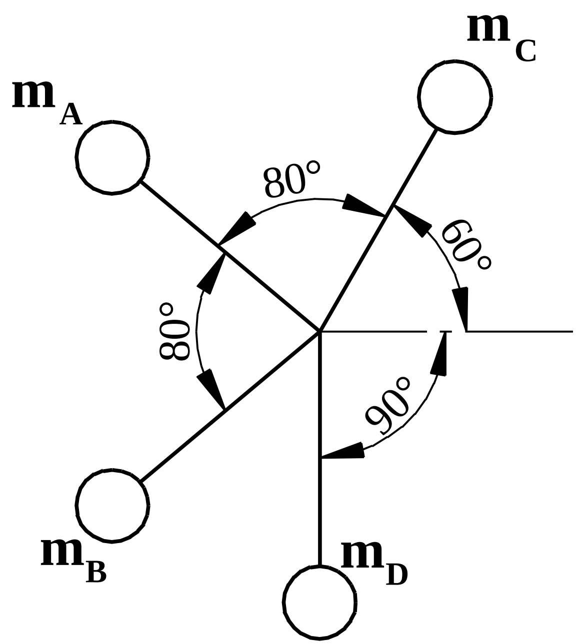




Plane	Mass m (kg)	Radius (r) mts	Force m r (kg-m)	Distance from R.P. (l) mts	Couple mrl kg-m^2
A	20	0.05	1	- 0.05	-0.05
P)*	m_L	r_L	$m_L r_L$	0	0
B	10	0.05	0.5	0.05	0.025
C	10	0.03	0.3	0.13	0.039
D	15	0.04	0.6	0.19	0.114
M	m_M	r_M	$m_M r_M$	0.23	$0.23 m_M r_M$



Couple Polygon

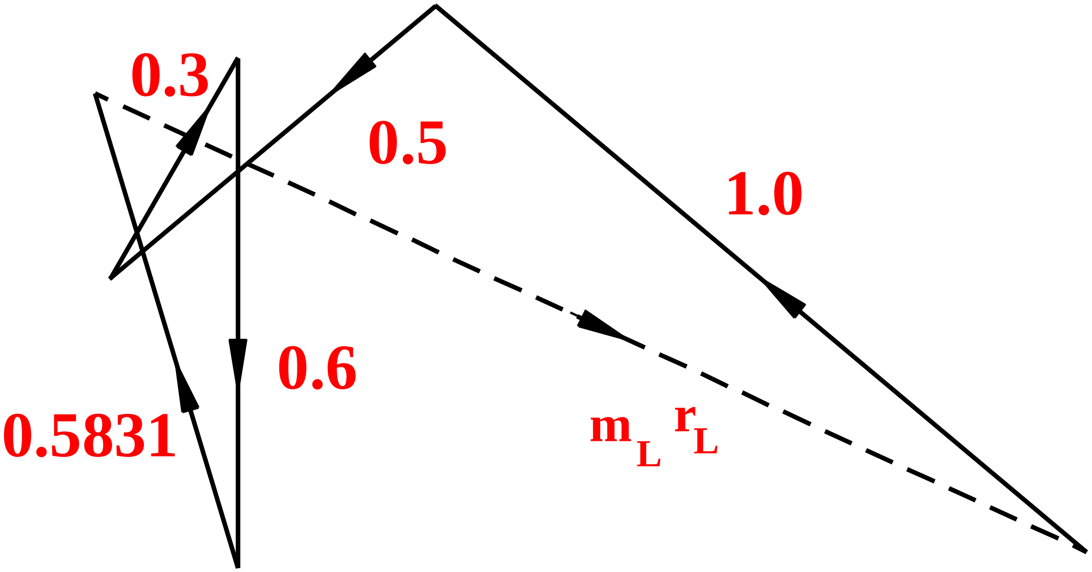


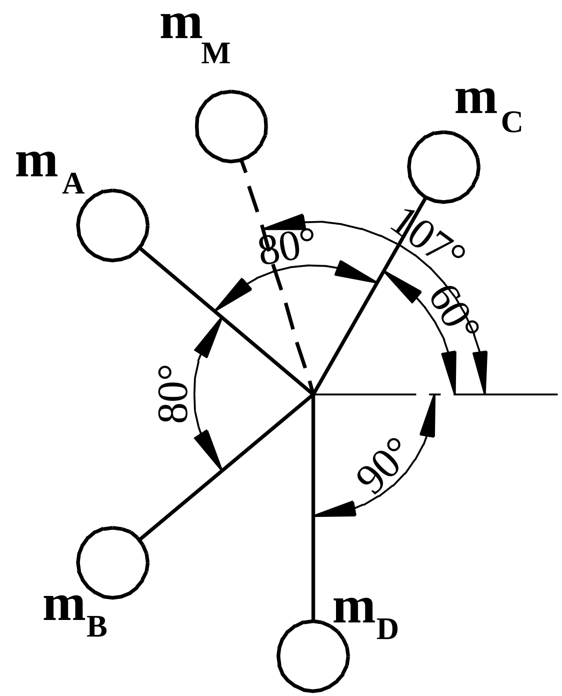
From the couple polygon,

$$m_M r_M = 0.5831\ kg\text{-}m^2, \theta_M = 106.74^0$$

Hence, dynamic force at the bearing M

$$= m_M r_M \omega^2 = 0.5831\left(\frac{2\pi \times 600}{60}\right)^2 = \mathbf{2302N}$$





Force Polygon

From the force polygon,

$$m_L r_L = 1.286 \text{ kg - m}, \theta_L = 335^0$$

Hence, dynamic force at the bearing L

$$= m_L r_L \omega^2 = 1.286 \left(\frac{2\pi \times 600}{60} \right)^2 = \mathbf{5077N}$$

Analytical Solution :

Note:As the couple table has only one unknown, it may be used to find the value of $m_M r_M$ & direction θ_M.

As the system of masses is balanced by itself, summation of horizontal & Vertical of components couples is individually equal to zero.

Sum of Horizontal components of couples = 0

i.e. $\sum C_H = 0 \Rightarrow \sum mrl\cos\theta = 0$

$$\begin{bmatrix} (-0.05\times\cos 140^0)+0+(0.025\cos 220^0)+(0.039\cos 60^0) \\ +(0.114\cos 270^0)+(0.23 m_M r_M \cos\theta_M) \end{bmatrix} = 0$$

$\therefore$ ***$m_M r_M \cos\theta_M = -0.168$……..(i)***

Sum of vertical components of couples = 0

i.e. $\sum C_V = 0 \Rightarrow \sum mrl\sin\theta = 0$

$$\begin{bmatrix} (-0.05\times\sin 140^0)+0+(0.025\sin 220^0)+(0.039\sin 60^0) \\ +(0.114\sin 270^0)+(0.23 m_M r_M \sin\theta_M) \end{bmatrix} = 0$$

$\therefore$ ***$m_M r_M \sin\theta_M = 0.5584$……..(ii)***

Squaring & adding (i) & (ii),

$$m_M r_M \sqrt{\cos^2\theta_M + \sin^2\theta_M} = \sqrt{(-0.168)^2 + (0.5584)^2}$$

$$\Rightarrow m_M r_M = 0.5831 \text{ kg-m.}$$

Hence, dynamic force at the bearing M

$$= m_M r_M \omega^2 = 0.5831\left(\frac{2\pi \times 600}{60}\right)^2 = \mathbf{2302N}$$

Angular Position of mass M (θ_M)

Dividing eqn (ii) by (i), we get,

$$\theta_M = \tan^{-1}\left(\frac{0.5584}{-0.168}\right) = -73.26^0$$

As numerator is +ve & denominator is -ve, the angle is in the Second quadrant.

Hence $\theta_M = (180 - 73.26) = 106.74^0$ measured ccw w.r.t. horizontal.

To find the value of $m_L r_L$

As the system of masses is balanced by itself, summation of horizontal & Vertical of components forces is individually equal to zero.

Sum of Horizontal components of forces = 0

$$\text{i.e.} \sum F_H = 0 \Rightarrow \sum mr\cos\theta = 0$$

$$\begin{bmatrix} (1\times\cos 140^0) + m_L r_L \cos\theta_L + (0.5\cos 220^0) + (0.3\cos 60^0) \\ +(0.6\cos 270^0) + (0.5831\cos 106.74^0) \end{bmatrix} = 0$$

$$\therefore m_L r_L \cos\theta_L = 1.167 \ldots\ldots\ldots (iii)$$

Sum of vertical components of forces = 0

$$\text{i.e.} \sum F_V = 0 \Rightarrow \sum mr\sin\theta = 0$$

$$\begin{bmatrix} (1\times\sin 140^0) + m_L r_L \sin\theta_L + (0.5\sin 220^0) + (0.3\sin 60^0) \\ +(0.6\sin 270^0) + (0.5831\sin 106.74^0) \end{bmatrix} = 0$$

$$\therefore m_L r_L \sin\theta_L = -0.5396 \ldots\ldots\ldots (iv)$$

Squaring & adding (iii) & (iv),

$$m_L r_L \sqrt{\cos^2\theta_L + \sin^2\theta_L} = \sqrt{(1.167)^2 + (-0.5396)^2}$$

$$\Rightarrow m_L r_L = 1.2857\ kg-m$$

Hence, dynamic force at the bearing L

$$= m_L r_L \omega^2 = 1.2857\left(\frac{2\pi \times 600}{60}\right)^2 = \mathbf{5076N}$$

Angular Position of mass L (θ_L)

Dividing eqn (ii) by (i), we get,

$$\theta_M = \tan^{-1}\left(\frac{-0.5396}{1.167}\right) = -24.81^0 \approx -25^0$$

As numerator is -ve & denominator is +ve, the angle is in the Fourth quadrant.

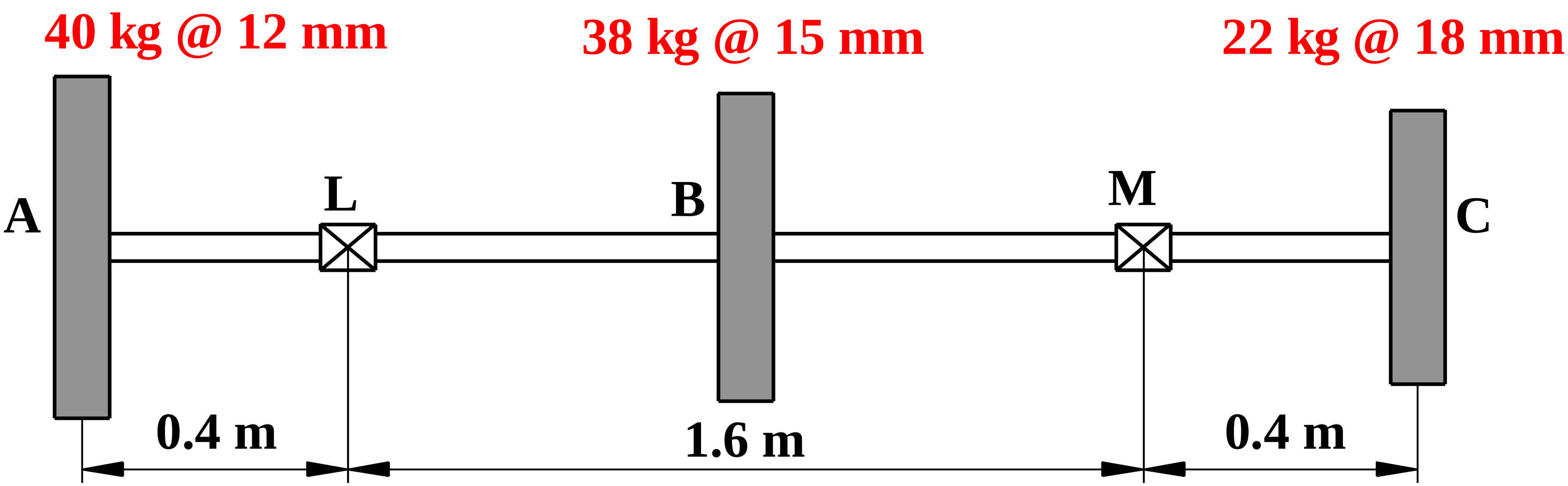
$$\therefore \theta_L = (360 - 25) = 335^0$$

Problem 5

A shaft supported in bearings 1.6 m apart projects 400 mm beyond bearings at each end. It carries three pulleys one at each end and one at the center of its length. The masses of the end pulleys are 40 kg and 22 kg and their eccentricities are 12 mm & 18 mm respectively from the shaft axis. The mass of center pulley is 38 kg at 15 mm radius. ***The pulleys are arranged in a manner that they give static balance***. Determine;

a) The relative angular positions of the pulleys
b) The dynamic forces developed on the bearings when the shaft rotates at 210 rpm.

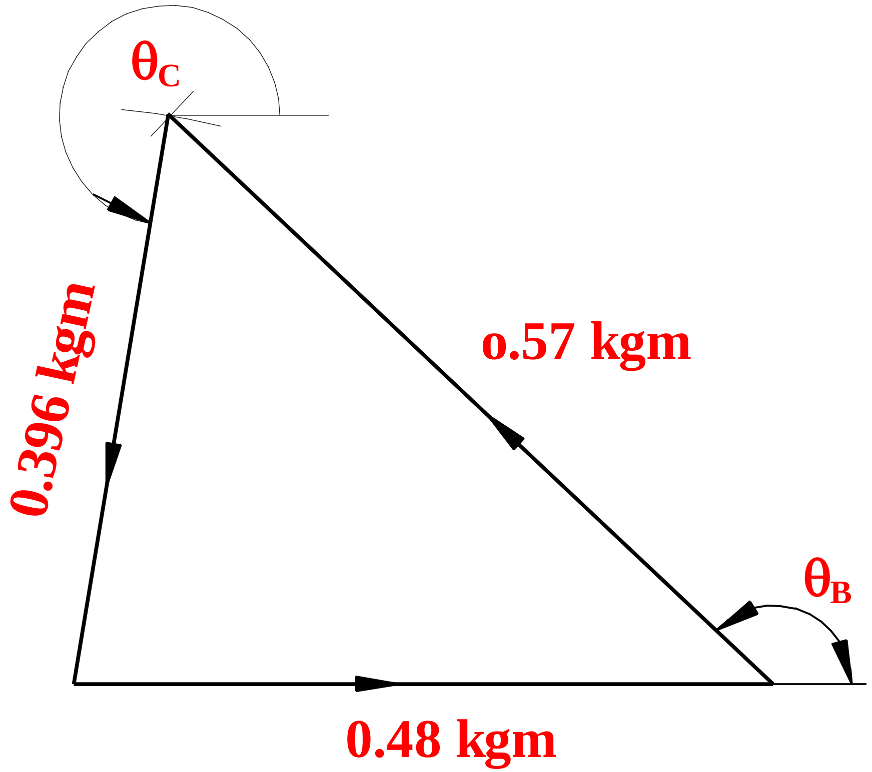
Space diagram



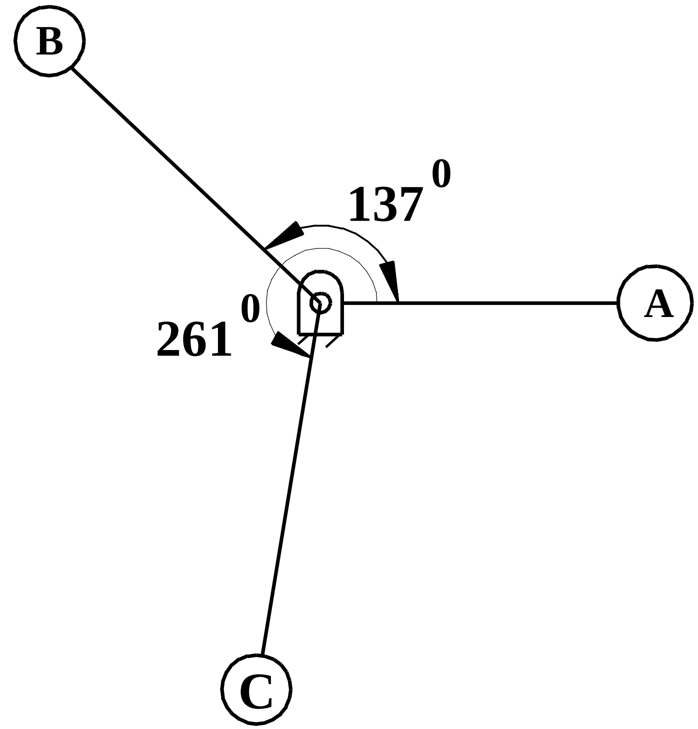
1. As the pulleys are arranged for static balance, the ***mr*** values of pulleys A, B & C must form a closed polygon, from which their relative angular positions can be obtained.

2. To find the dynamic reaction at bearings, assume balancing masses in the bearing planes and find their ***mr*** values. Then the dynamic forces at the bearings will be $mr\omega^2$.

Plane	Mass m (kg)	Radius (r) mts	Force m r (kg-m)	Distance from R.P. (l) mts	Couple mrl kg-m^2
A	40	0.012	0.48	- 0.4	-0.192
P)*	m_L	r_L	$m_L r_L$	0	0
B	38	0.015	0.57	0.8	0.456
M	m_M	r_M	$m_M r_M$	1.6	$1.6 m_M r_M$
C	22	0.018	0.396	2	0.792

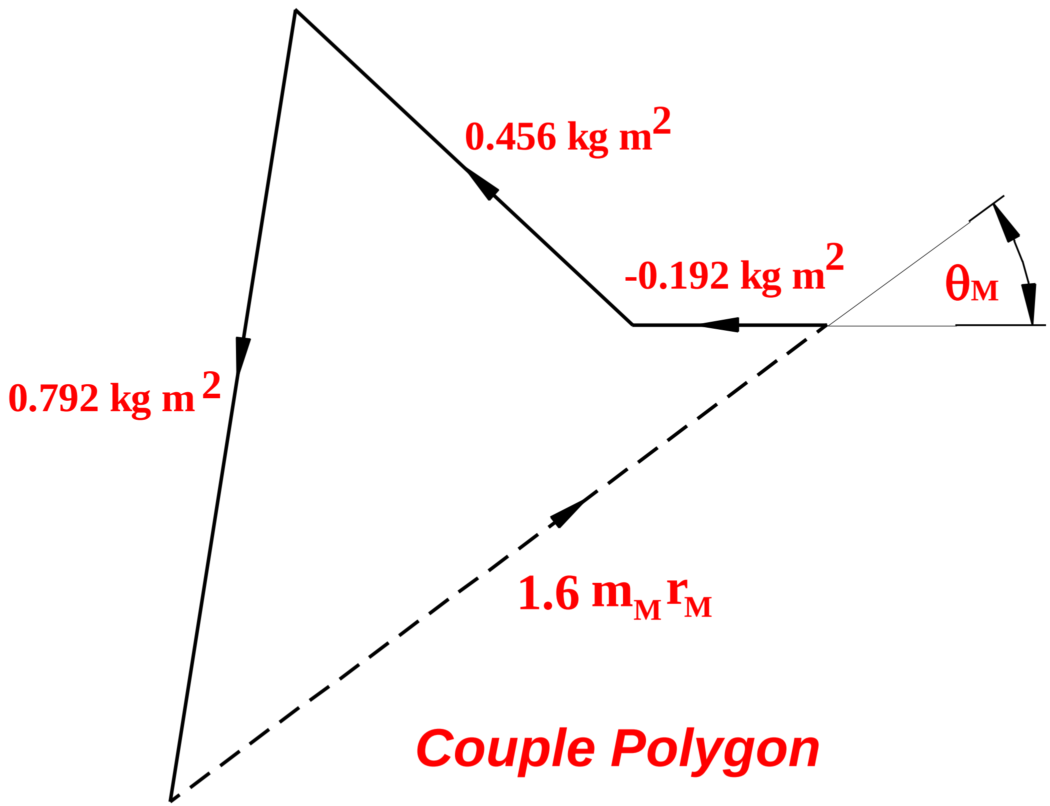


Static Force Polygon
(To obtain angles between the masses)

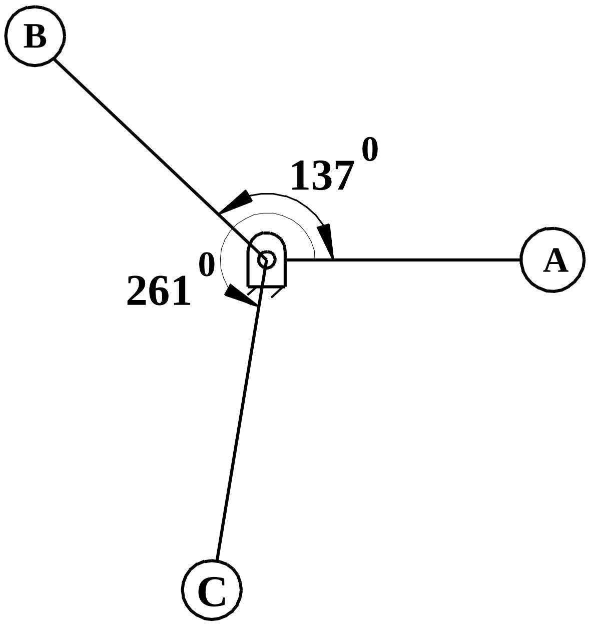


Angular Positions of masses

From the force polygon,
$\theta_B = 137^0$, $\theta_C = 261^0$ w.r.t A



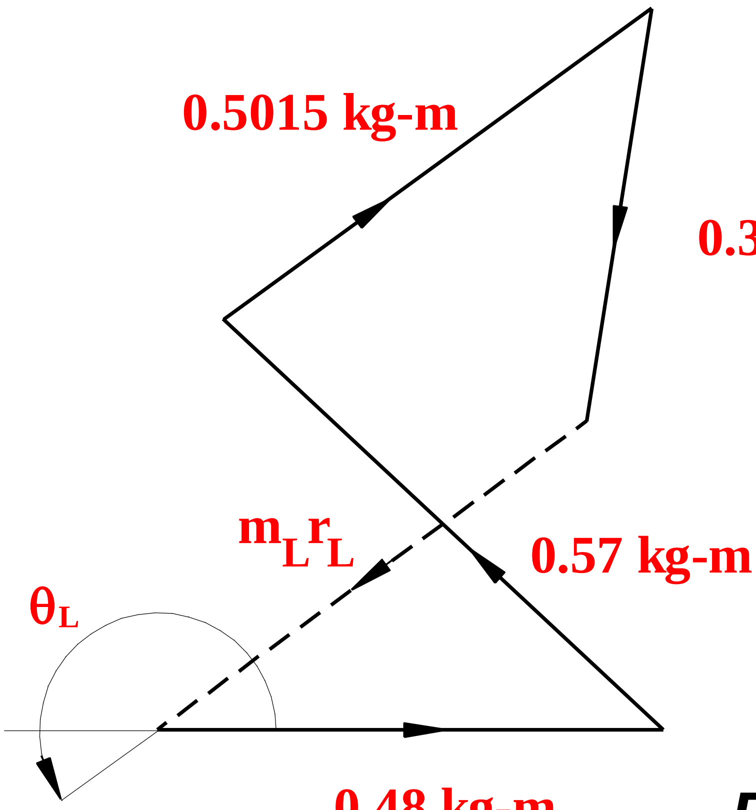
Couple Polygon

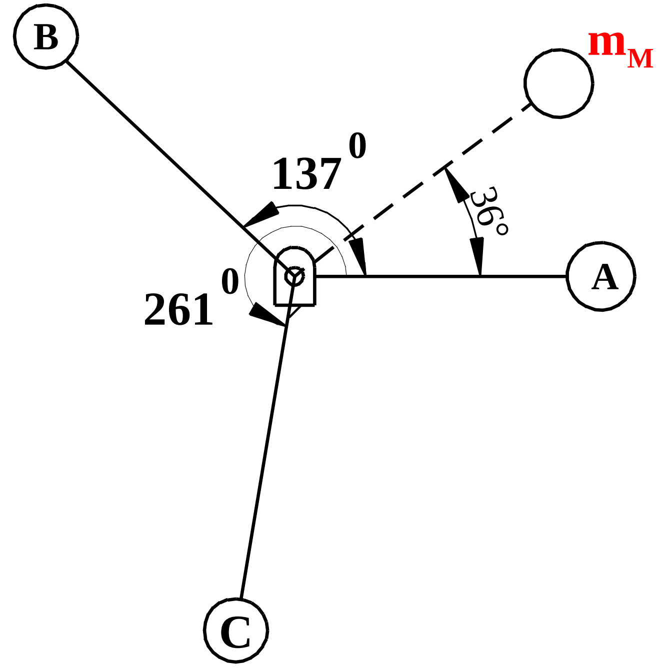


From the couple polygon, $\theta_M = 36^0$, $m_M r_M = 0.5015$ kgm.

Hence the dynamic reaction at the bearing M

$$=m_M r_M \omega^2 = 0.5015\left(\frac{2\times\pi\times 210}{60}\right)^2 = \mathbf{242.53N}$$





Force Polygon

From the couple polygon, $\theta_L = 324^0$, $m_L r_L = 0.501$ kgm.

Hence the dynamic reaction at the bearing L

$$= m_L r_L \omega^2 = 0.501\left(\frac{2\times\pi\times210}{60}\right)^2 = \mathbf{242.3N}$$