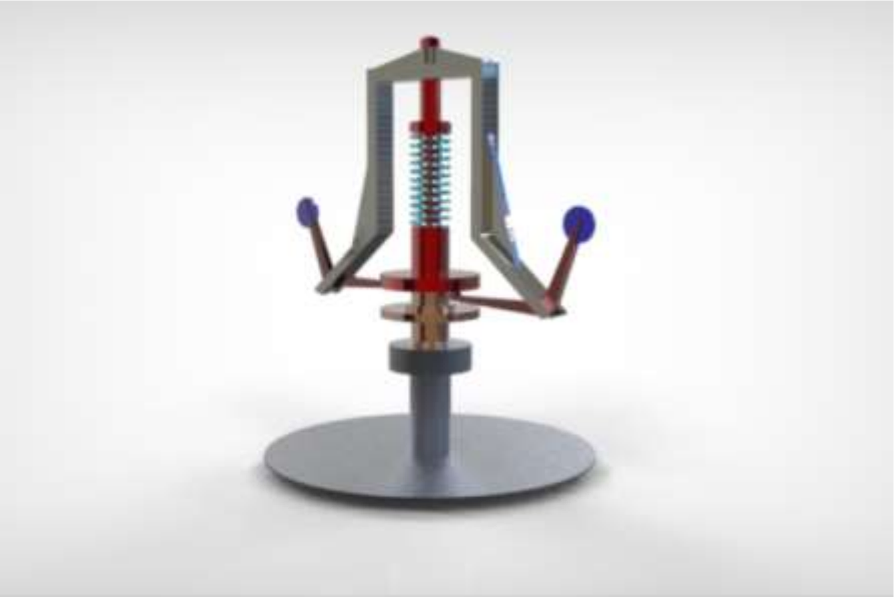
Governors

PORTER GOVERNOR



HARTNELL GOVERNOR

Governors

- The function of a governor is to regulate the mean speed of an engine, when there are variations in the load.
- When the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.
- The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

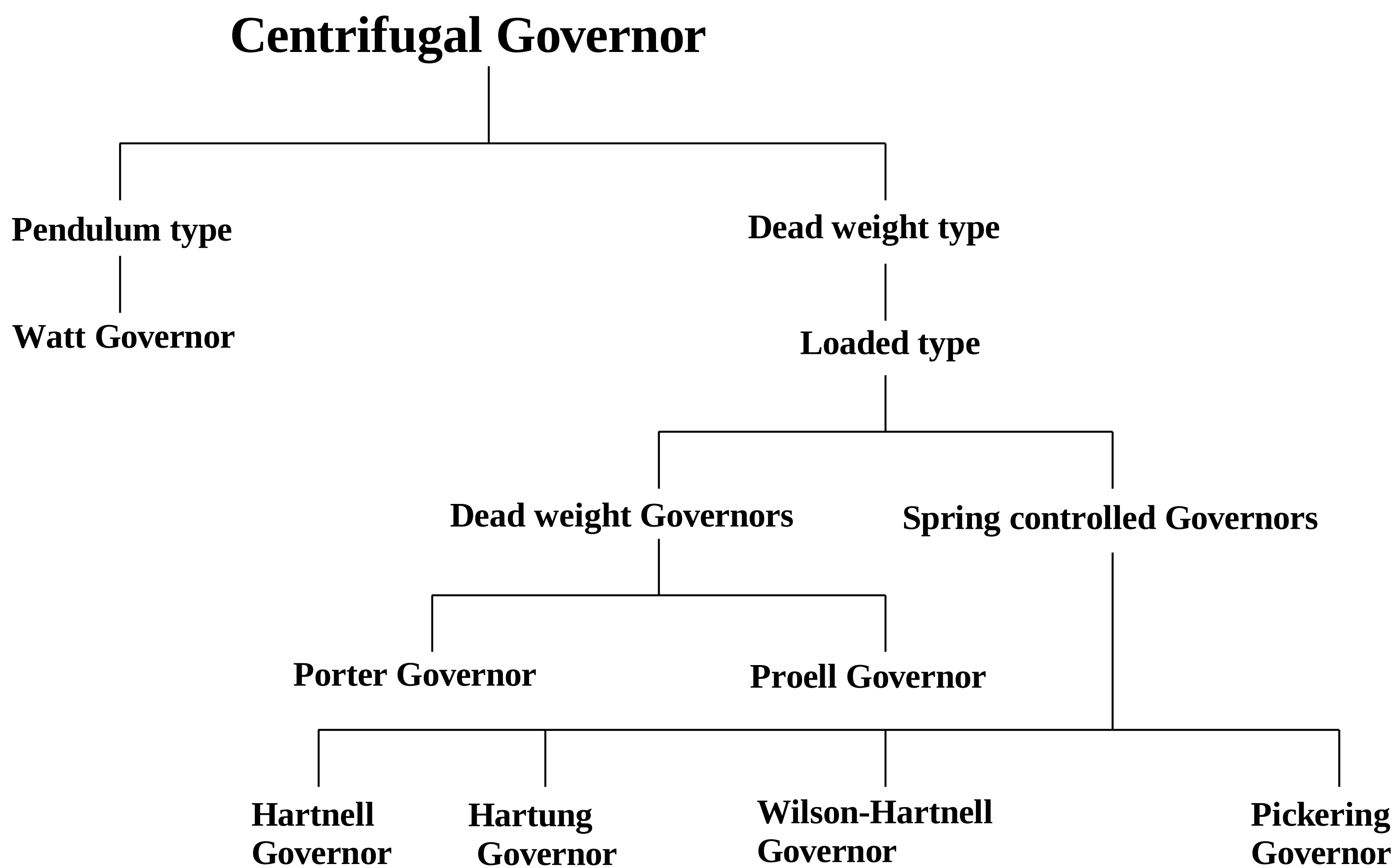
Governors

- The function of a flywheel in an engine is entirely different from that of a governor.
- It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load.
- The varying demand for power is met by the governor regulating the supply of working fluid.

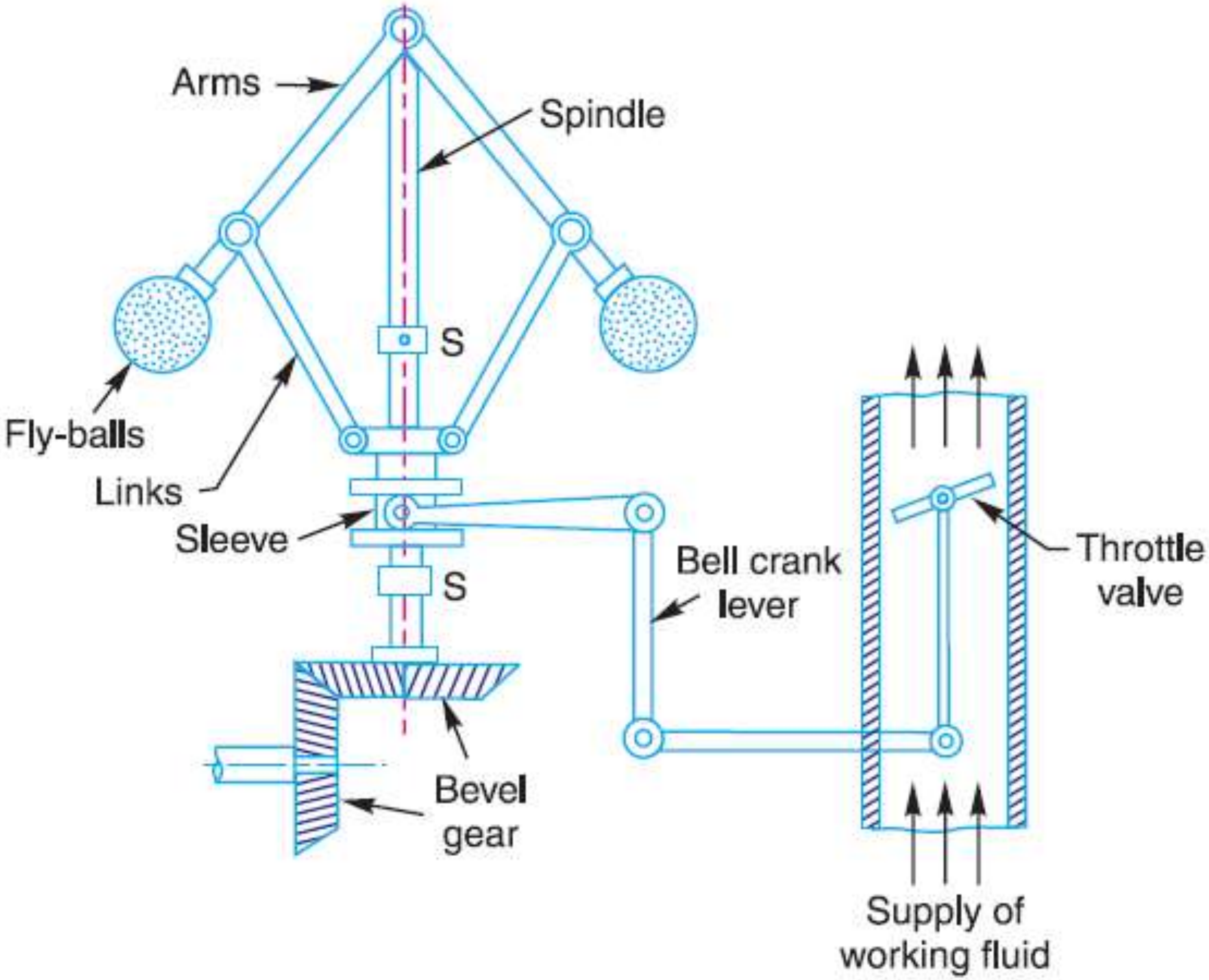
Types of Governors

The governors may broadly be classified as;

1. Centrifugal governors 2. Inertia governors.



Governors



Arms
Spindle
S
Fly-balls
Links
Sleeve
S
Bell crank
lever
Throttle
valve
Bevel
gear
Supply of
working fluid

Centrifugal Governor

- The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force.***
- It consists of two equal masses, which are attached to the arms as shown in fig. These masses are known as ***governor masses* or *fly balls*.**
- The masses revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis.
- The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle ; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases.

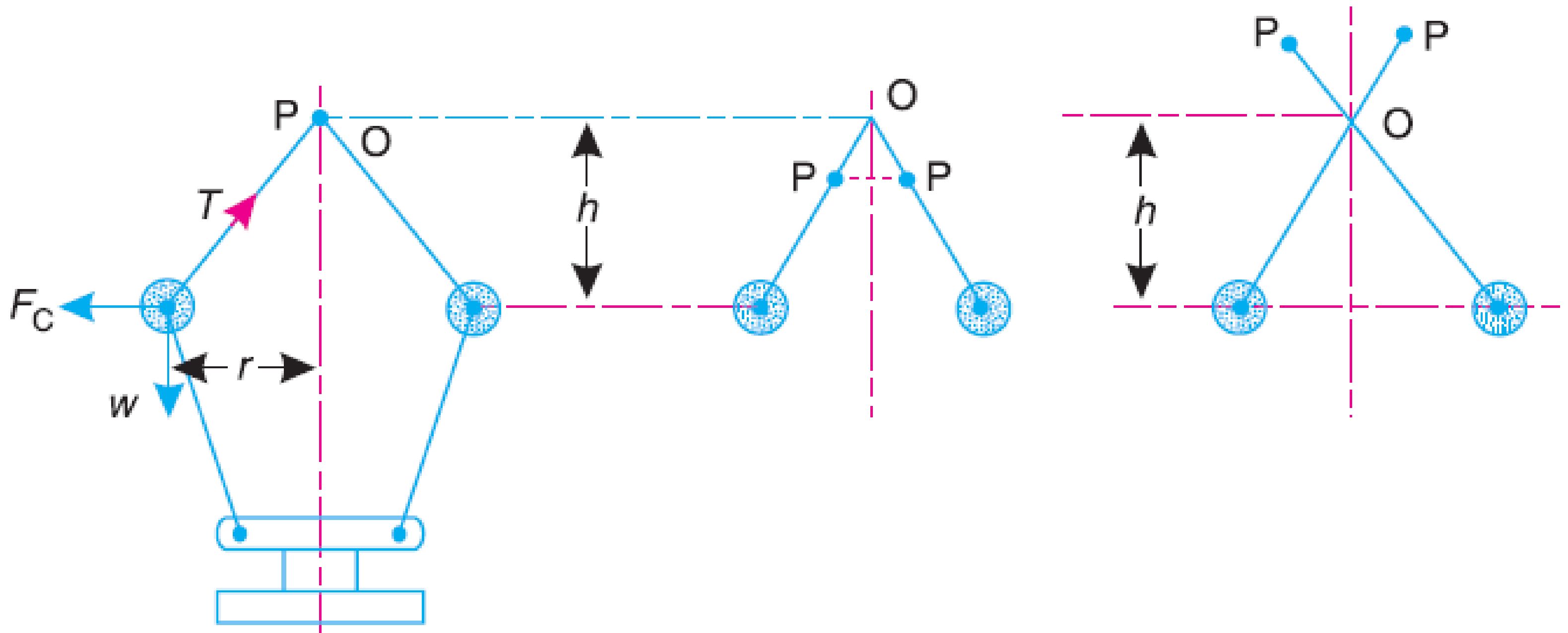
Centrifugal Governor

- When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards.
- The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load.
- When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Terms Used in Governors

The following terms are used in governors;

1. *Height of a governor*. It is the vertical distance from the center of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by *h.*



(*a*) (*b*) (*c*)

Terms Used in Governors

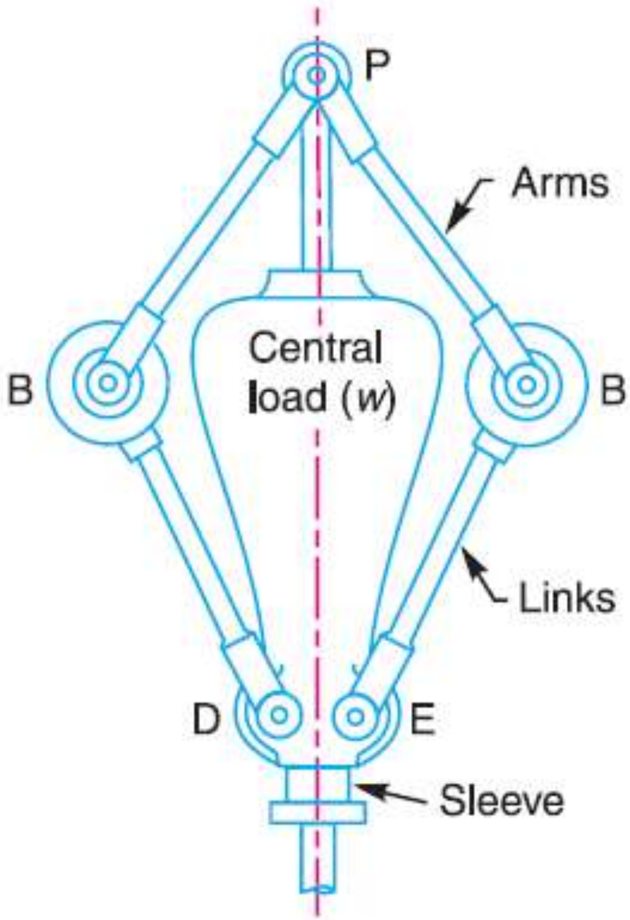
2. ***Equilibrium speed*****.** It is the speed at which the governor balls, arms ,etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. ***Mean equilibrium speed*****.** It is the speed at the mean position of the balls or the sleeve.

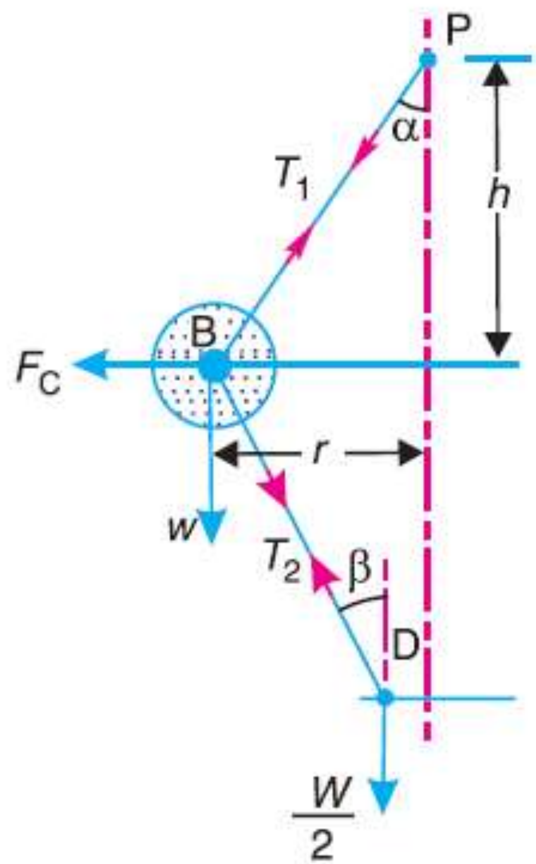
4. ***Maximum and minimum equilibrium speeds*****.** The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

5. ***Sleeve lift*****.** It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Porter Governor



(a)



(b)

Equilibrium Speed of a Porter Governor

Let m = Mass of each ball in kg,

w = Weight of each ball in newton = mg

M = Mass of the central load in kg

W = Weight of the central load in newton = Mg

r = Radius of rotation in meters

h = Height of governor in meters

N = Speed of the spindle in rpm

ω = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s

F_C = Centrifugal force acting on the ball in newton = $m\omega^2 r$

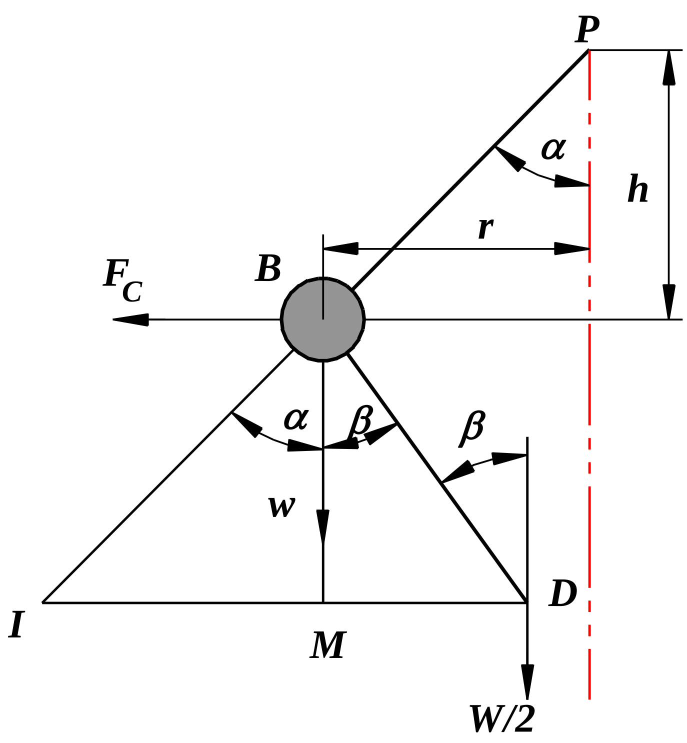
α = Angle of inclination upper links to the vertical

β = Angle of inclination of the lower links to the vertical.

Equilibrium Speed of a Porter Governor

(by Instantaneous center method)

The instantaneous center I lies at the point of intersection of PB produced and a line through D perpendicular to the spindle axis, as shown in fig. Taking moments about the point I,



$$F_c \times BM = (w \times IM) + \left(\frac{W}{2} \times ID\right)$$

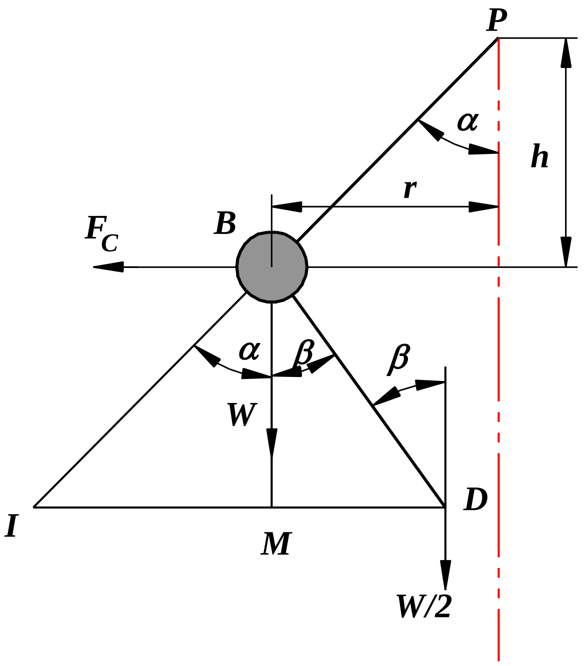
$$= (mg \times IM) + \left(\frac{Mg}{2} \times ID\right)$$

$$\therefore F_c = mg\left(\frac{IM}{BM}\right) + \frac{Mg}{2}\left(\frac{ID}{BM}\right)$$

$$= mg\left(\frac{IM}{BM}\right) + \frac{Mg}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= mg(\tan\alpha) + \frac{Mg}{2}(\tan\alpha + \tan\beta)$$

Equilibrium Speed of a Porter Governor



Dividing throughout by $\tan\alpha$,

$$\frac{F_c}{\tan\alpha} = mg + \frac{Mg}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)$$

$$= mg + \frac{Mg}{2}(1+q) \cdots\cdots \left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

But $F_c = m\omega^2 r$ and $\tan\alpha = \dfrac{r}{h}$

$$\therefore\ m\omega^2 r \times \left(\frac{h}{r}\right) = mg + \frac{Mg}{2}(1+q)$$

$$\Rightarrow \omega^2 = \left(\frac{mg + \frac{Mg}{2}(1+q)}{mh}\right)$$

Equilibrium Speed of a Porter Governor

Substituting $\omega = \dfrac{2\pi N}{60}$,

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \left(\frac{M}{2}\right)(1+q)}{m} \times \frac{g}{h}$$

$$N^2 = \frac{m + \left(\frac{M}{2}\right)(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \left(\frac{M}{2}\right)(1+q)}{m} \times \frac{895}{h}$$

Note :

If all links are equal & pivoted on the axis of rotation,

$\tan\beta = \tan\alpha \Rightarrow q = 1$. Hence the equation becomes;

$$N^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h}$$

Effect of friction at sleeve:

- When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.
- If *F* = Frictional force acting on the sleeve in newton, then the equations for equilibrium speed may be written as;

$$N^2 = \frac{mg + \left(\frac{Mg \pm F}{2}\right)(1+q)}{mg} \times \frac{895}{h}$$

If all links are equal & pivoted on the axis of rotation,

$\tan\beta = \tan\alpha \Rightarrow q = 1$. Hence the equation becomes;

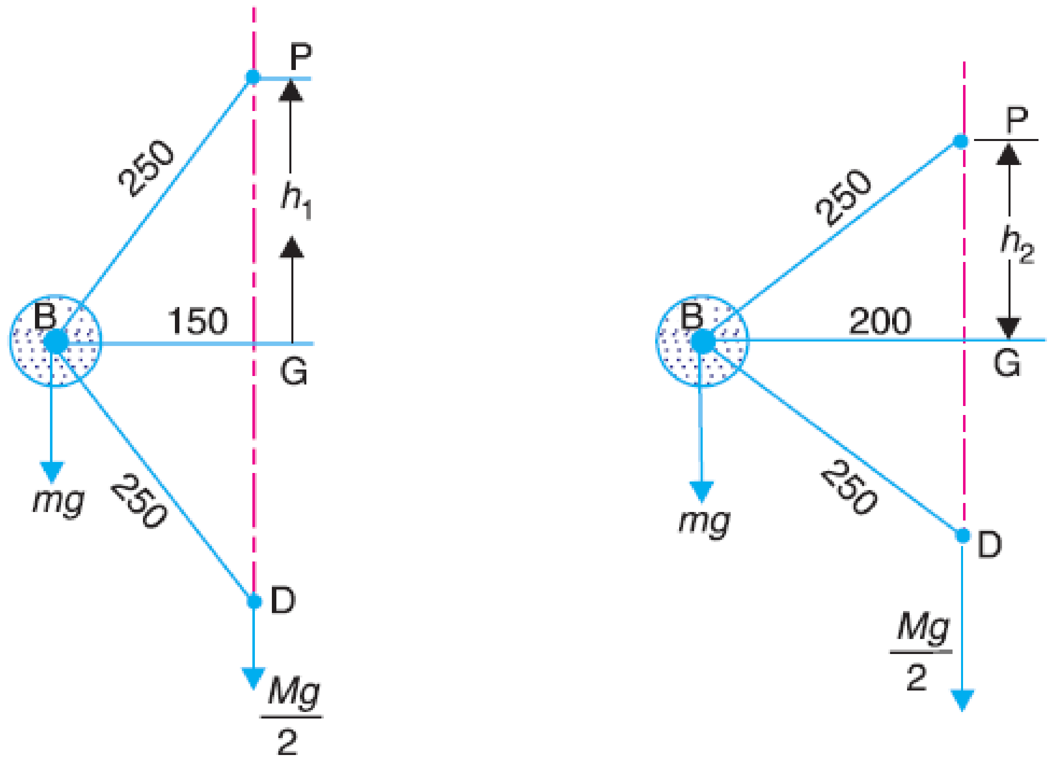
$$N^2 = \frac{mg + (Mg \pm F)}{mg} \times \frac{895}{h}$$

Problem 1

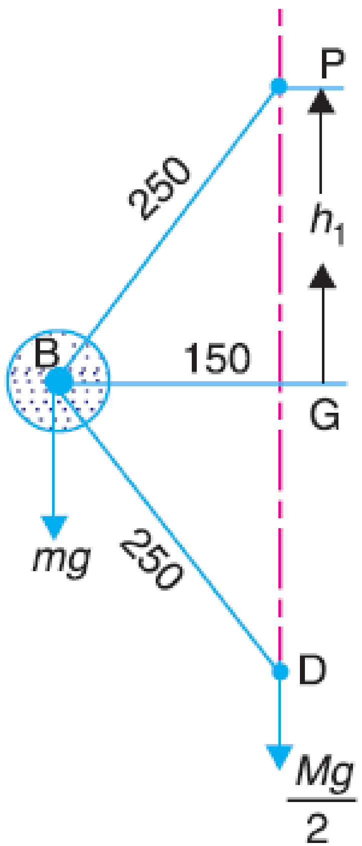
A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 15 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Given : *length of upper links* = length of lower links =250 mm = 0.25 m ;
m = 5 kg ; M = 15 kg ; r_1 = 150 mm = 0.15m; r_2 = 200 mm = 0.2 m

The minimum and maximum positions of the governor are shown in Figs (*a*) and (*b*) respectively.



(*a*) Minimum position. (*b*) Maximum position.



(a) Minimum position.

Minimum speed position :

Let N_1 be the minimum speed.

From fig (a), height of the governor

$$h_1 = \sqrt{0.25^2 - 0.15^2} = 0.2m$$

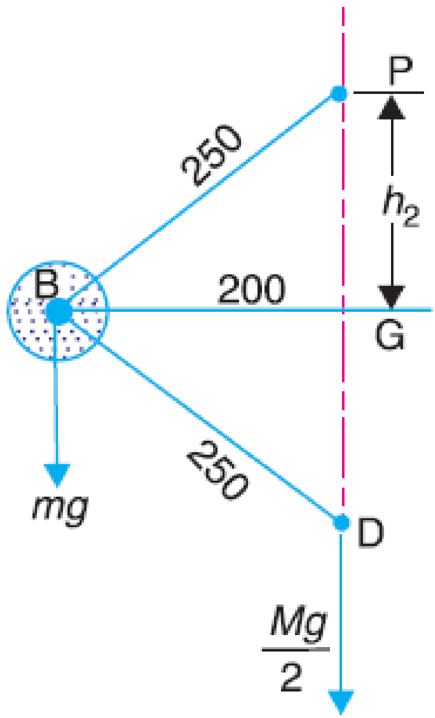
Since all links are equal & pivoted on the axis of rotation, $\tan\beta = \tan\alpha \Rightarrow q = 1.$

Hence the equation becomes;

$$N_1^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h}$$

$$N_1^2 = \left(\frac{5+15}{5}\right) \times \frac{895}{0.2} \Rightarrow N_1 = 133.8 \text{ rpm}$$

Maximum speed position :



(b) Maximum position.

Let N_2 be the maximum speed.

From fig (b), height of the governor

$$h_2 = \sqrt{0.25^2 - 0.2^2} = 0.15m$$

$$\therefore N_2^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h}$$

$$N_2^2 = \left(\frac{5+15}{5}\right) \times \frac{895}{0.15}$$

$\Rightarrow$ $N_2 = 154.5\ rpm$

Hence the range of speed $= (N_2 - N_1) = (154.5 - 133.8)$ $= 20.7\ rpm$

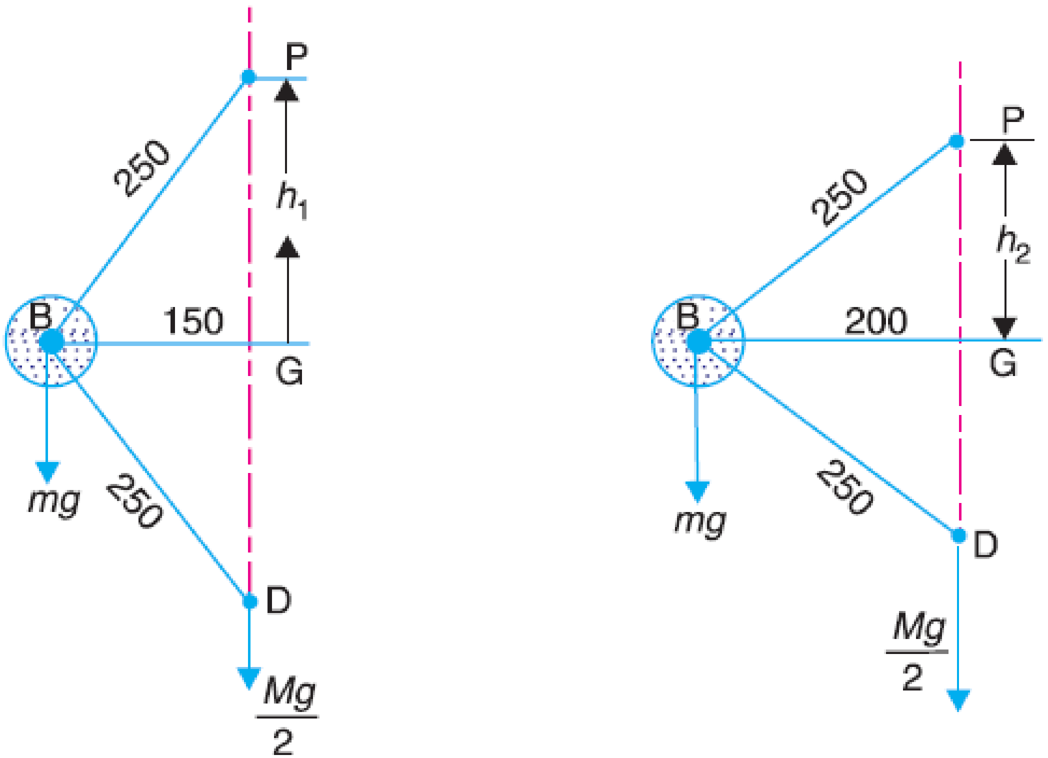
Problem 2

The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor.

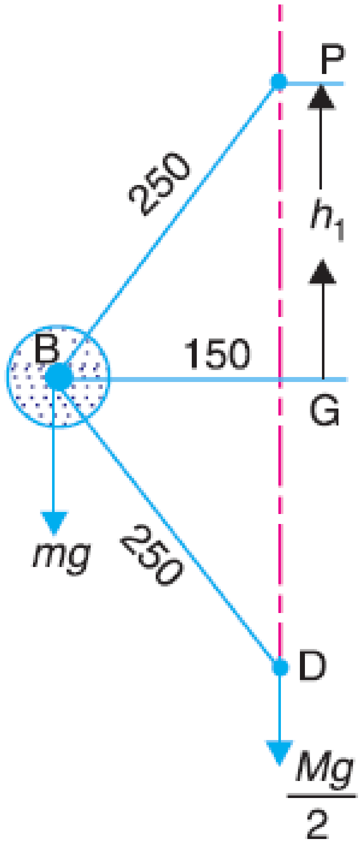
If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

Given : *length of upper links* = length of lower links =250 mm = 0.25 m ;
m = 5 kg ; M = 30 kg ; r_1 = 150 mm = 0.15m; r_2 = 200 mm = 0.2 m

The minimum and maximum positions of the governor are shown in Figs (*a*) and (*b*) respectively.



(*a*) Minimum position. (*b*) Maximum position.



(a) Minimum position.

Minimum speed position :

Let N_1 be the minimum speed.

From fig (a), height of the governor

$$h_1 = \sqrt{0.25^2 - 0.15^2} = 0.2m$$

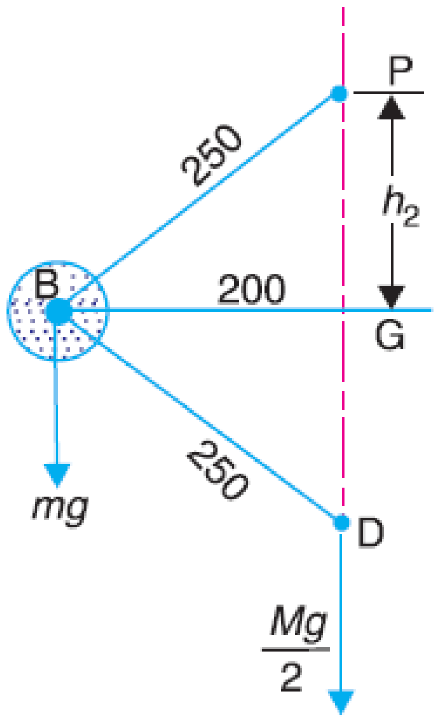
Since all links are equal & pivoted on the axis of rotation, $\tan\beta = \tan\alpha \Rightarrow q = 1.$

Hence the equation becomes;

$$N_1^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h}$$

$$N_1^2 = \left(\frac{5+30}{5}\right) \times \frac{895}{0.2} \Rightarrow N_1 = 177\ rpm$$

Maximum speed position :



(*b*) Maximum position.

Let N_2 be the maximum speed.

From fig (b), height of the governor

$$h_2 = \sqrt{0.25^2 - 0.2^2} = 0.15m$$

$$\therefore N_2^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h}$$

$$N_2^2 = \left(\frac{5+30}{5}\right) \times \frac{895}{0.15}$$

$$\Rightarrow N_2 = 204.4 \ rpm$$

Hence the range of speed $= (N_2 - N_1) = (204.4 - 177)$ **= 27.4 rpm**

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (*F*) acts upwards and the minimum speed is given by

$$N_1^2 = \frac{mg + (Mg - F)}{mg} \times \frac{895}{h_1}$$

$$= \frac{(5\times 9.81) + (30\times 9.81 - 20)}{5\times 9.81} \times \frac{895}{0.2} \Rightarrow \mathbf{N_1 = 172\ rpm}$$

Also when the sleeve moves upwards, the friction force (*F*) acts downwards and the maximum speed is given by

$$N_2^2 = \frac{mg + (Mg + F)}{mg} \times \frac{895}{h_2}$$

$$= \frac{(5\times 9.81) + (30\times 9.81 + 20)}{5\times 9.81} \times \frac{895}{0.15} \Rightarrow \mathbf{N_2 = 210\ rpm}$$

$$\text{Hence the range of speed} = (N_2 - N_1) = (210 - 172) = \mathbf{38\ rpm}$$

Problem 3

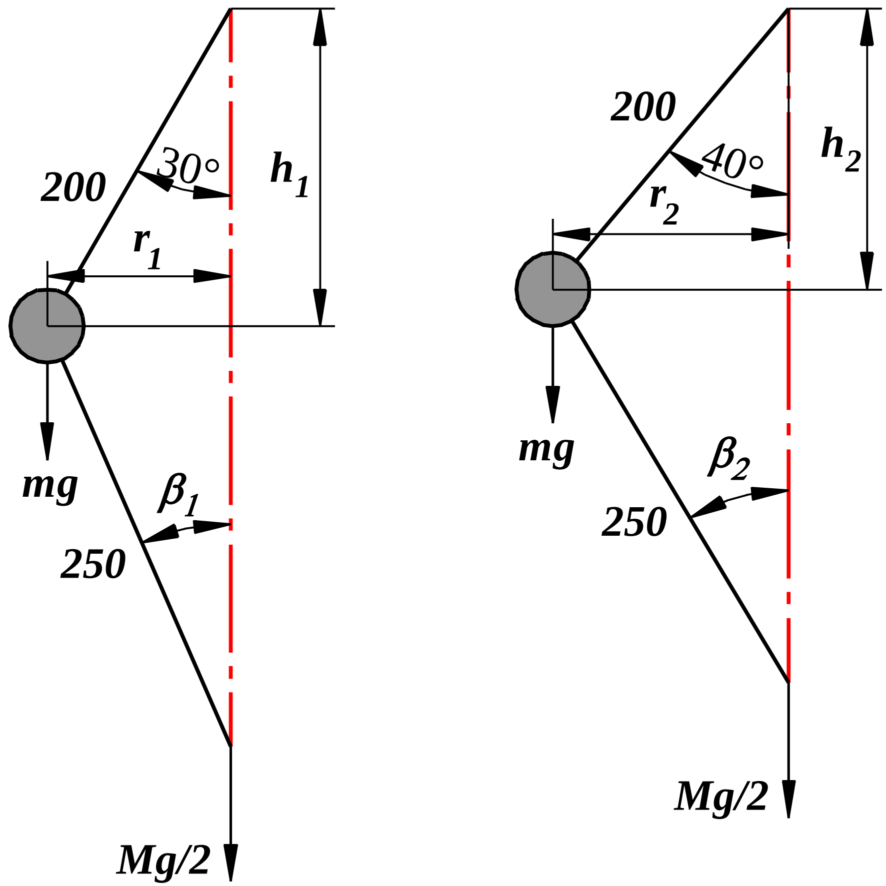
In an engine governor of the Porter type, the upper and lower arms are 200mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the frictional resistance equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, find, taking friction into account, range of speed of the governor.

Given : length of upper links =200 mm=0.2 m

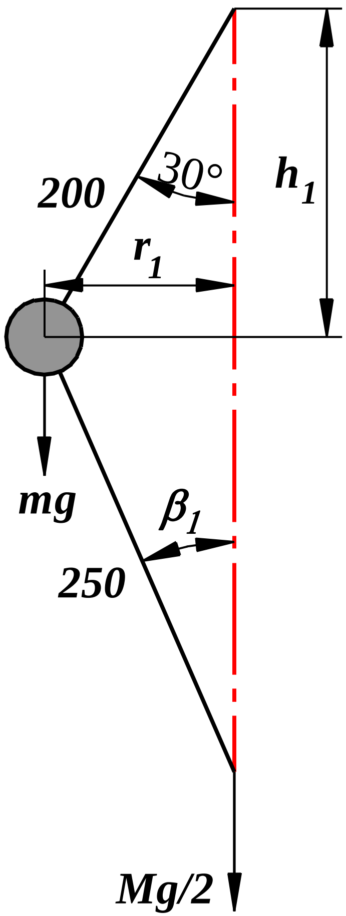
length of lower links =250 mm = 0.25 m

m = 2 kg ; M = 15 kg ; F=25N, $\alpha_1 = 30^0$; $a_2 = 40^0$

The minimum and maximum positions of the governor are shown in Figs (*a*) and (*b*) respectively.



Minimum Speed position ***Maximum Speed position***



Minimum Speed position

Minimum speed position : (sleeve is moving down)

$$N_1^2 = \frac{mg + \left(\frac{Mg - F}{2}\right)(1 + q_1)}{mg} \times \frac{895}{h_1}$$

From the fig, $r_1 = 0.2\sin 30^0 = 0.1m$

$$h_1 = 0.2\cos 30^0 = 0.1732m$$

$$\tan\alpha_1 = \tan 30^0 = 0.5774$$

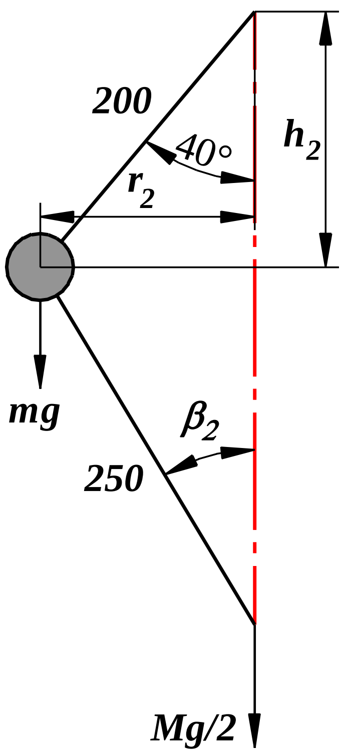
$$\sin\beta_1 = \frac{0.1}{0.25} = 0.4 \Rightarrow \tan\beta_1 = 0.4364$$

$$\therefore q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.4364}{0.5774} = 0.756$$

Substituting the above values,

$$N_1^2 = \frac{2\times 9.81 + \left(\frac{15\times 9.81 - 25}{2}\right)(1 + 0.756)}{2\times 9.81} \times \left(\frac{895}{0.1732}\right)$$

$$\Rightarrow N_1 = 183rpm$$



Maximum Speed position

Maximum speed position : (sleeve is moving up)

$$N_2^2 = \frac{mg + \left(\frac{Mg+F}{2}\right)(1+q_2)}{mg} \times \frac{895}{h_2}$$

From the fig,

$$r_2 = 0.2\sin 40^0 = 0.1268\ m,\ \ h_2 = 0.2\cos 40^0 = 0.1532m$$

$$\tan\alpha_2 = \tan 40^0 = 0.8391,\ \ \sin\beta_2 = \frac{0.1268}{0.25} = 0.5072$$

$$\Rightarrow \tan\beta_2 = 0.5885$$

$$\therefore q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.5885}{0.8391} = 0.7013$$

Substituting the above values,

$$N_2^2 = \frac{2\times 9.81 + \left(\frac{15\times 9.81+25}{2}\right)(1+0.7013)}{2\times 9.81} \times \left(\frac{895}{0.1532}\right)$$

$$\Rightarrow \mathbf{N_2 = 223\ rpm}$$

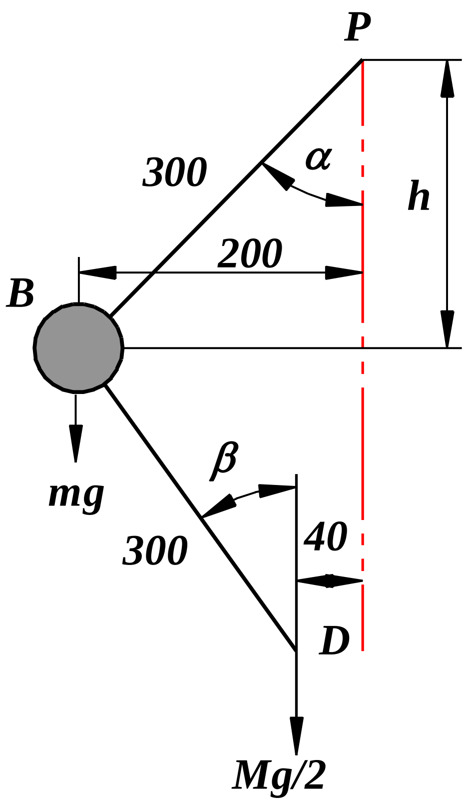
Hence the range of speed $= (N_2 - N_1) = (223 - 183)$ **= 40 rpm**

Problem 4

The arms of a Porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200 mm. If the friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position ?

Equilibrium speed when r = 200 mm : (neglecting friction)

$$N = \frac{mg + \left(\frac{Mg}{2}\right)(1+q)}{mg} \times \frac{895}{h}$$



From the fig, $h = \sqrt{0.3^2 - 0.2^2} = 0.2236m$

$$\tan\alpha = \frac{r}{h} = \frac{0.2}{0.2236} = 0.894$$

$$\sin\beta = \frac{0.2 - 0.04}{0.3} = 0.5333 \Rightarrow \tan\beta = 0.63$$

$$\therefore q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.894} = 0.705$$

Substituting the above values,

$$N^2 = \frac{10\times 9.81 + \left(\frac{70\times 9.81}{2}\right)(1+0.705)}{10\times 9.81} \times \left(\frac{895}{0.2236}\right)$$

$\Rightarrow$ **N = 167 rpm**

Speed range when friction at the sleeve is equivalent of 20 N of load :

Here, for the same radius of rotation, for **impending upward or downward motion of sleeve**, speed range can be calculated. *r, q & h* values remain same.

Equilibrium speed : (for impending downward motion of sleeve)

$$N_1^2 = \frac{mg + \left(\frac{Mg - F}{2}\right)(1+q)}{mg} \times \frac{895}{h}$$

Substituting the above values,

$$N_1^2 = \frac{10\times 9.81 + \left(\frac{70\times 9.81 - 20}{2}\right)(1+0.705)}{10\times 9.81} \times \left(\frac{895}{0.2236}\right)$$

$$\Rightarrow N_1 = 164.9 \text{ rpm}$$

Equilibrium speed : ***(for impending upward motion of sleeve)***

$$N_2^2 = \frac{mg + \left(\frac{Mg + F}{2}\right)(1+q)}{mg} \times \frac{895}{h}$$

Substituting the above values,

$$N_2^2 = \frac{10\times 9.81 + \left(\frac{70\times 9.81 + 20}{2}\right)(1+0.705)}{10\times 9.81} \times \left(\frac{895}{0.2236}\right)$$

$\Rightarrow$ ***N_1 = 169.1 rpm***

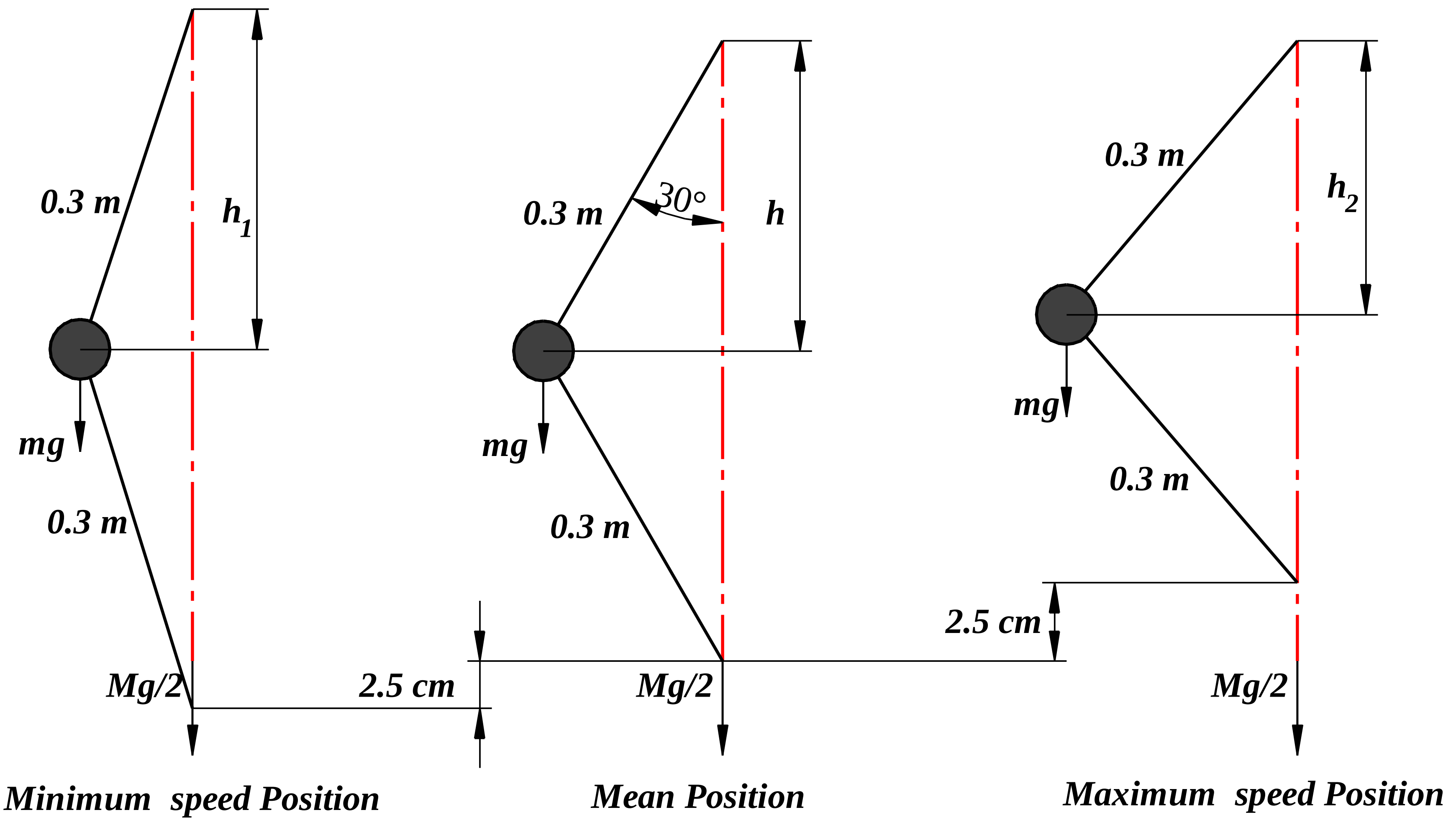
Hence the range of speed $= (N_2 - N_1) = (169.1 - 164.9)$ ***= 4.2 rpm***

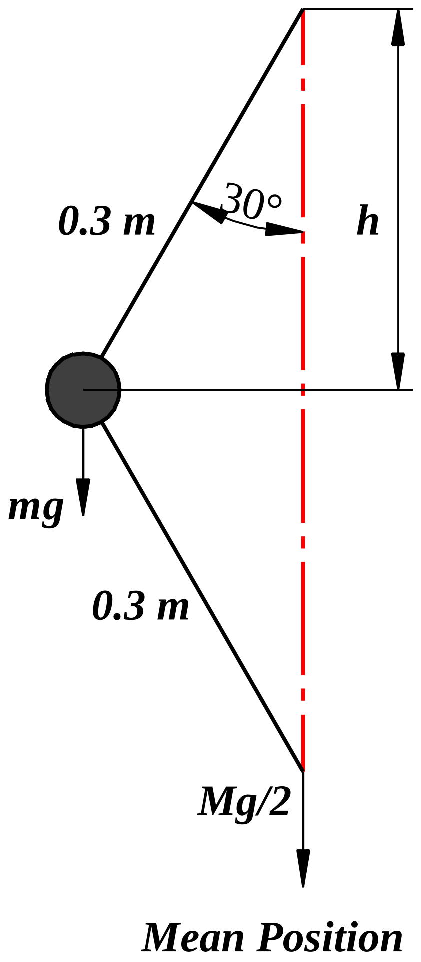
Problem 5

The arms of a porter governor are each 30 cm long and are pivoted on the governor axis. Mass of each ball is 2 kg. At the mean speed of 150 rpm, the arm makes 30^0 with the vertical. Determine the central load and the sensitivity of the governor if the sleeve movement is ±2.5 cm.

Given : length of upper links = length of lower links =300 mm=0.25 m

m = 2 kg ; M = ? α = 30^0 ; N = 150 rpm





Mean speed position :

Here, $N = 150\ rpm$.

From fig, height of the governor

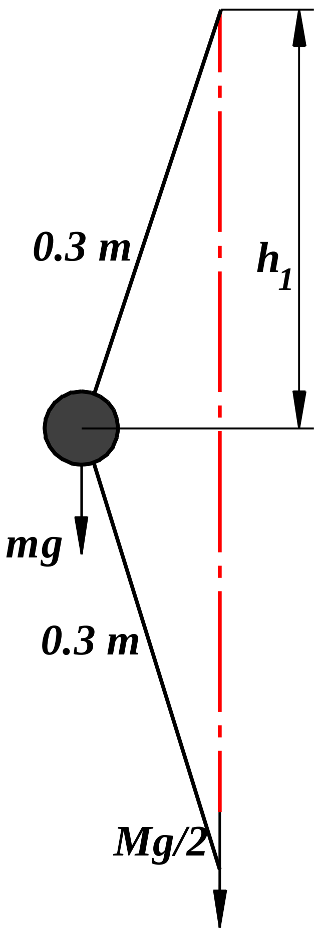
$$h = 0.3\cos 30^0 = 0.2598m$$

Here, friction is neglected & $\alpha = \beta \Rightarrow$ q=1

$$\therefore\ N^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h}$$

$$150^2 = \left(\frac{2+M}{2}\right) \times \frac{895}{0.2598}$$

$\Rightarrow$ **$M = 11.06\ kg$**



Minimum Speed Position

Minimum speed position :

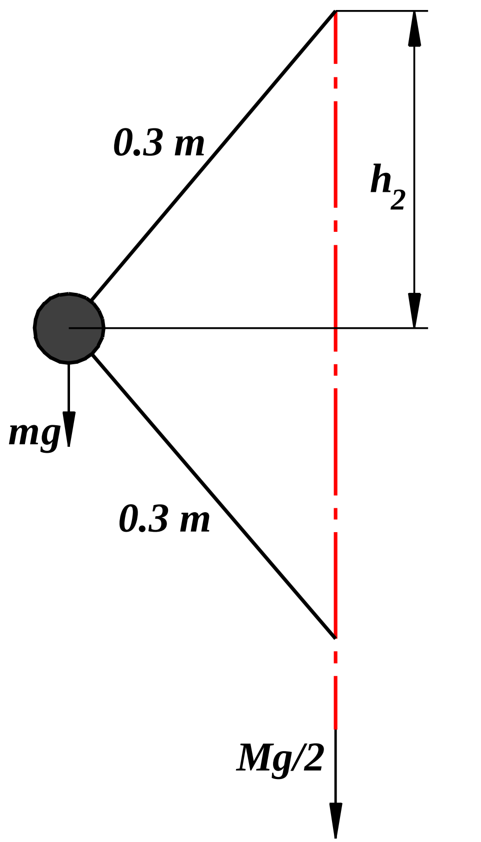
(After a drop of 2.5cm of sleeve)

Due to symmetry, height of governor $h_1 = h + \left(\frac{x}{2}\right)$

$$\Rightarrow \; h_1 = h + \left(\frac{x}{2}\right) = 0.2598 + \left(\frac{2.5\times10^{-3}}{2}\right) = 0.2723m$$

$$\therefore \; N_1^2 = \left(\frac{m+M}{m}\right)\times\frac{895}{h_1}$$

$$N_1^2 = \left(\frac{2+11.06}{2}\right)\times\frac{895}{0.2723} \Rightarrow N_1 = 146.44 \text{ rpm}$$



Maximum Speed Position

Maximum speed position :

(After a raise of 2.5cm of sleeve)

Due to symmetry, height of governor $h_2 = h - \left(\frac{x}{2}\right)$

$$\Rightarrow \; h_2 = h - \left(\frac{x}{2}\right) = 0.2598 - \left(\frac{2.5\times10^{-3}}{2}\right) = 0.2473m$$

$$\therefore \; N_2^2 = \left(\frac{m+M}{m}\right)\times\frac{895}{h_2}$$

$$N_2^2 = \left(\frac{2+11.06}{2}\right)\times\frac{895}{0.2473} \Rightarrow N_2 = 153.68 \text{ rpm}$$

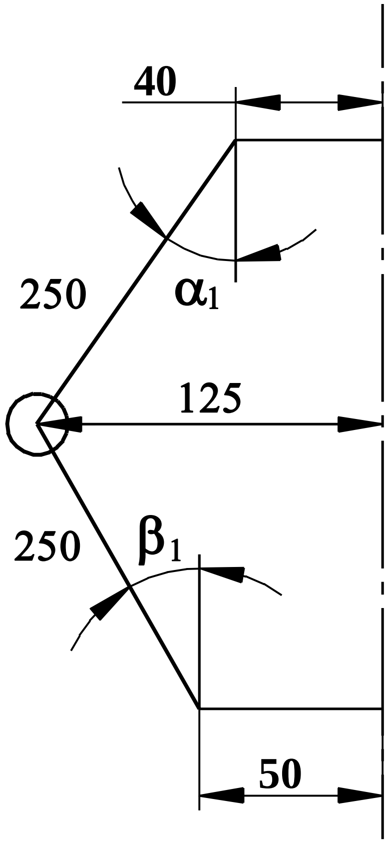
$$\therefore \text{Sensitivity} = \frac{\text{Range of speed}}{\text{Mean speed}} = \frac{2(N_2 - N_1)}{(N_2 + N_1)} = \frac{2(153.68 - 146.44)}{(153.68 + 146.44)} = 4.83\%$$

Problem 6

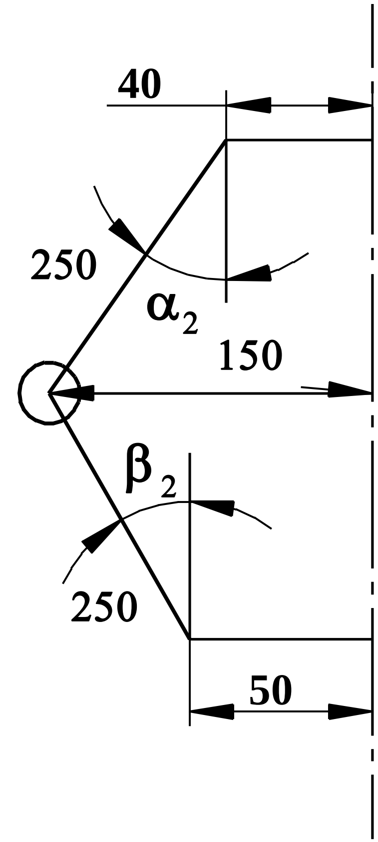
Each arm of a porter governor is 250 mm long. The upper arms and the lower arms are pivoted to links 40 mm & 50 mm respectively from the axis of rotation. Each ball has a mass of 5 kg & the sleeve mass is 50 kg. The force of friction at the sleeve is 40 N. Determine the range of speed of the governor for extreme radii of 125 mm & 150 mm.

Given : length of upper links = length of lower links =250 mm=0.25 m

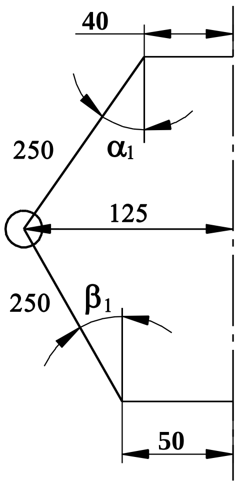
m = 5 kg ; M = 50 kg ; F=40N, r_1 = 125 mm; r_2 = 150 mm



Minimum radius position



Maximum radius position



Minimum radius position

(i) ***When r = 125 mm***, $\sin\alpha_1 = \left(\dfrac{125-40}{250}\right) = 0.34$

$$\Rightarrow \alpha_1 = 19.88^0 \;\therefore \tan\alpha_1 = 0.362,\; \sin\beta_1 = \left(\frac{125-50}{250}\right) = 0.3$$

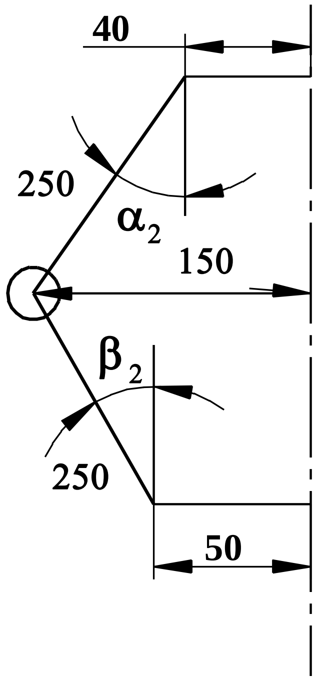
$$\Rightarrow \beta_1 = 17.46^0 \;\therefore \tan\beta_1 = 0.3145$$

$$\Rightarrow q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.3145}{0.362} = 0.87$$

$$mr_1\omega_1^2 = \tan\alpha_1\left[mg + \frac{Mg-f}{2}(1+q_1)\right]$$

$$\Rightarrow 5\times0.125\times\omega_1^2 = 0.362\left[5\times9.81 + \frac{(50\times9.81)-40}{2}(1+0.87)\right]$$

$\therefore \omega_1 = 16.5 rad/\sec$ Or ***N_1 = 157.6rpm***



Maximum radius position

(ii) ***When r = 150 mm***, $\sin\alpha_2 = \left(\dfrac{150-40}{250}\right) = 0.44$

$$\Rightarrow \alpha_2 = 26.1^0 \quad \therefore \tan\alpha_2 = 0.49, \quad \sin\beta_2 = \left(\frac{150-50}{250}\right) = 0.4$$

$$\Rightarrow \beta_2 = 23.58^0 \quad \therefore \tan\beta_2 = 0.4364$$

$$\therefore q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.4364}{0.49} = 0.891$$

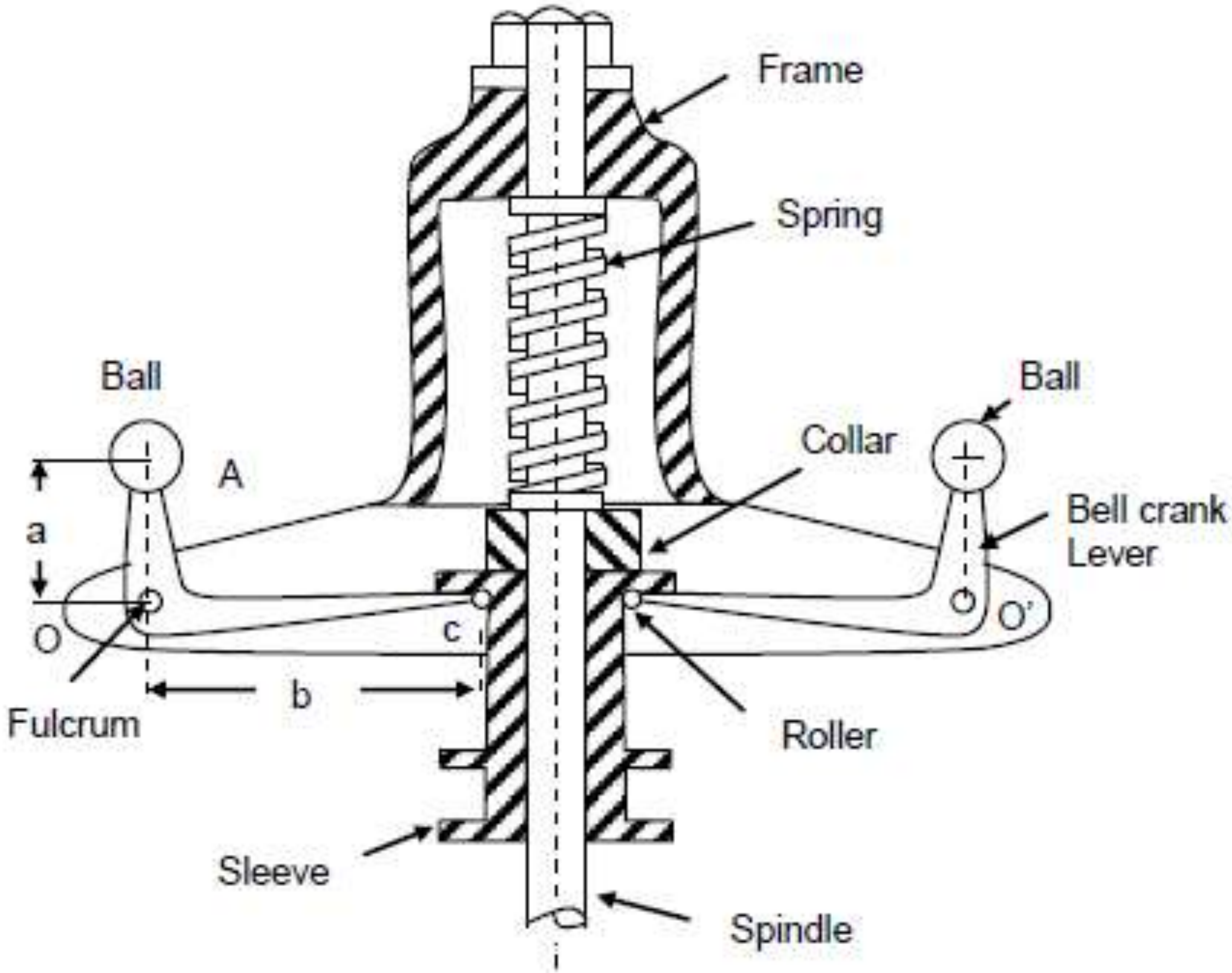
$$mr_2\omega_2^2 = \tan\alpha_2\left[mg + \frac{Mg-f}{2}(1+q_2)\right]$$

$$\Rightarrow 5\times 0.15\times\omega_2^2 = 0.49\left[5\times 9.81 + \frac{(50\times 9.81)+40}{2}(1+0.891)\right]$$

$\therefore \omega_2 = 18.97 rad/\sec$ Or $\boldsymbol{N_2}$ ***= 181.1rpm***

Range of speed = N_2 - N_1 = 23.5 rpm.

Hartnell Governor



Hartnell Governor

- The Hartnell governor is shown in Figure.
- The two bell crank levers have been provided which can have rotating motion about fulcrums *O* and *O*'.
- One end of each bell crank lever carries a ball and a roller at the end of other arm. The rollers make contact with the sleeve.
- The frame is connected to the spindle. A helical spring is mounted around the spindle between frame and sleeve. With the rotation of the spindle, all these parts rotate.
- With the increase of speed, the radius of rotation of the balls increases and the rollers lift the sleeve against the spring force. With the decrease in speed, the sleeve moves downwards.
- The movement of the sleeve are transferred to the throttle of the engine through linkages.

Expression for stiffness of spring of a Hartnell Governor

Let r_1 = Minimum radius of rotation of ball from spindle axis, in m,

r_2 = Maximum radius of rotation of ball from spindle axis, in m,

S_1 = Spring force exerted on sleeve at minimum radius, in N,

S_2 = Spring force exerted on sleeve at maximum radius, in N,

m = Mass of each ball, in kg,

M = Mass of sleeve, in kg,

N_1 = Minimum speed of governor at minimum radius, in rpm,

N_2 = Maximum speed of governor at maximum radius, in rpm,

ω_1 and ω_2 = Corresponding minimum and maximum angular velocities, in r/s,

F_1 = Centrifugal force corresponding to minimum speed= $m\omega_1^2 r_1$,

F_2 = Centrifugal force corresponding to maximum speed = $m\omega_2^2 r_2$

s = Stiffness of spring or rate of spring (N/m)

r = Distance of fulcrum O from the governor axis or radius of rotation,

a = Length of ball arm of bell-crank lever, i.e. distance OA, and

b = Length of sleeve arm of bell-crank lever, i.e. distance OC.

Expression for stiffness of spring of a Hartnell Governor

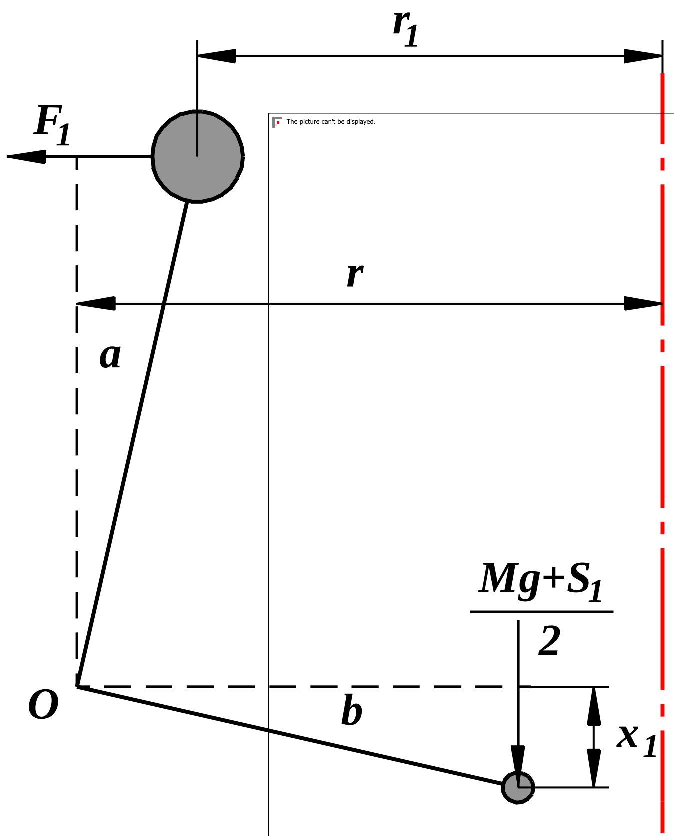


Fig (a)

Considering the position of the ball at radius 'r_1', as shown in Figure (a) and taking moments of all forces about 'O' (*neglecting moment due to weight of balls as it is negligible compared to spring forces & neglecting friction & obliquity of arms)*

$$F_1 \times a = \left(\frac{Mg + S_1}{2} \right) \times b \cdots\cdots (i)$$

where $F_1 = m\omega_1^2 r_1$ (r_1 must be in meters)

$$\& \ \omega_1 = \frac{2\pi N_1}{60}$$

Expression for stiffness of spring of a Hartnell Governor

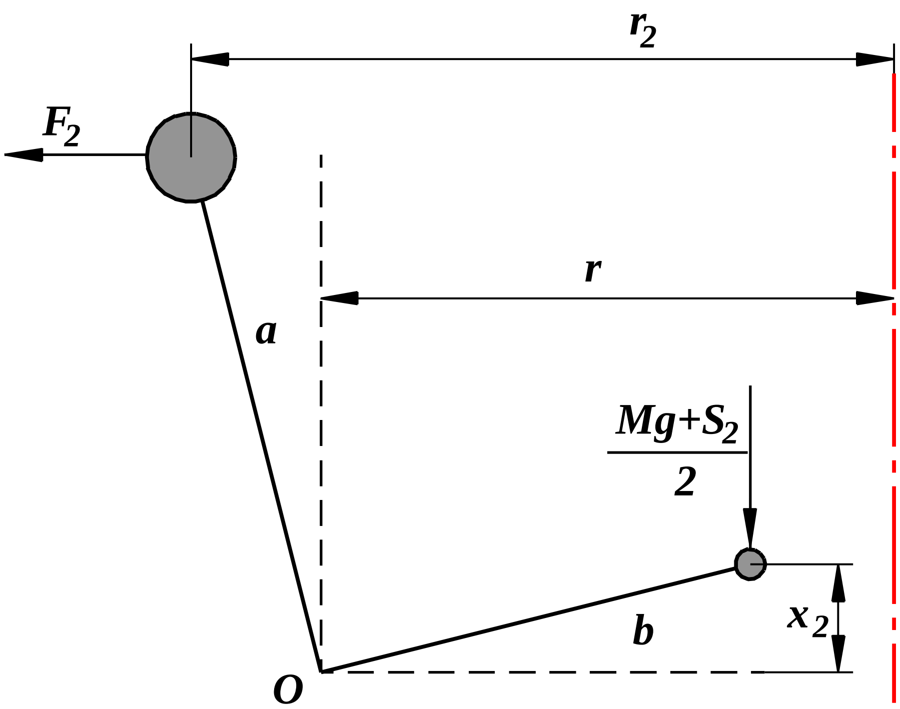


Fig (b)

Similarly, considering the position of the ball at radius ‘r_2’, as shown in Figure (b) and taking moments of all forces about ‘O’

$$F_2 \times a = \left(\frac{Mg + S_2}{2} \right) \times b \cdots\cdots (ii)$$

$$\text{where } F_2 = m\omega_2^2 r_2 \quad (\text{r}_2 \text{ must be in meters})$$

$$\& \ \omega_2 = \frac{2\pi N_2}{60}$$

Expression for stiffness of spring of a Hartnell Governor

Subtracting (i) from (ii),

$$(F_2 - F_1) \times a = \left(\frac{S_2 - S_1}{2} \right) \times b$$

$$\Rightarrow (F_2 - F_1) = \left(\frac{S_2 - S_1}{2} \right) \frac{b}{a} \cdots\cdots (iii)$$

From similar triangles (neglecting obliquity of arms)

$$\frac{x_1}{b} = \frac{r - r_1}{a} \Rightarrow x_1 = \left(\frac{r - r_1}{a} \right) b \quad \& \quad \frac{x_2}{b} = \frac{r_2 - r}{a} \Rightarrow x_2 = \left(\frac{r_2 - r}{a} \right) b$$

But Total sleeve lift $x = (x_1 + x_2) =$ Deformation of spring

$$x = \left(\frac{r - r_1}{a} \right) b + \left(\frac{r_2 - r}{a} \right) b \Rightarrow \boldsymbol{x = (r_2 - r_1) \frac{b}{a}}$$

Expression for stiffness of spring of a Hartnell Governor

Stiffness of spring :

$$s = \frac{\text{Change in spring force}}{\text{deformation of spring}} = \left(\frac{S_2 - S_1}{x}\right)$$

$$\Rightarrow (S_2 - S_1) = s \times x = s \times (r_2 - r_1)\frac{b}{a}, \text{Substituting for } (S_2 - S_1) \text{ in } (iii),$$

$$(F_2 - F_1) = \left(\frac{s \times (r_2 - r_1)}{2}\right)\frac{b^2}{a^2} \Rightarrow \mathbf{s = 2\frac{a^2}{b^2}\frac{(F_2 - F_1)}{(r_2 - r_1)}}$$

Note :

For any intermediate radius of rotation 'r' between max & min values, the centrifugal force F_C may be found by interpolation.

$$i.e.\ F = F_1 + \left(\frac{F_2 - F_1}{r_2 - r_1}\right) \times (r - r_1) \quad \textbf{OR} \quad F = F_2 - \left(\frac{F_2 - F_1}{r_2 - r_1}\right) \times (r_2 - r)$$

Problem 1

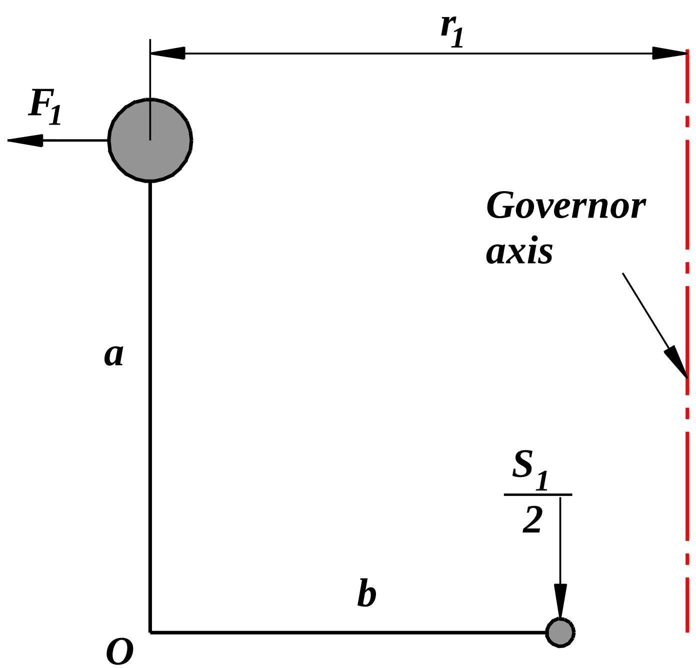
A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 rpm and 310 rpm for a sleeve lift of 15mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed.

Determine :

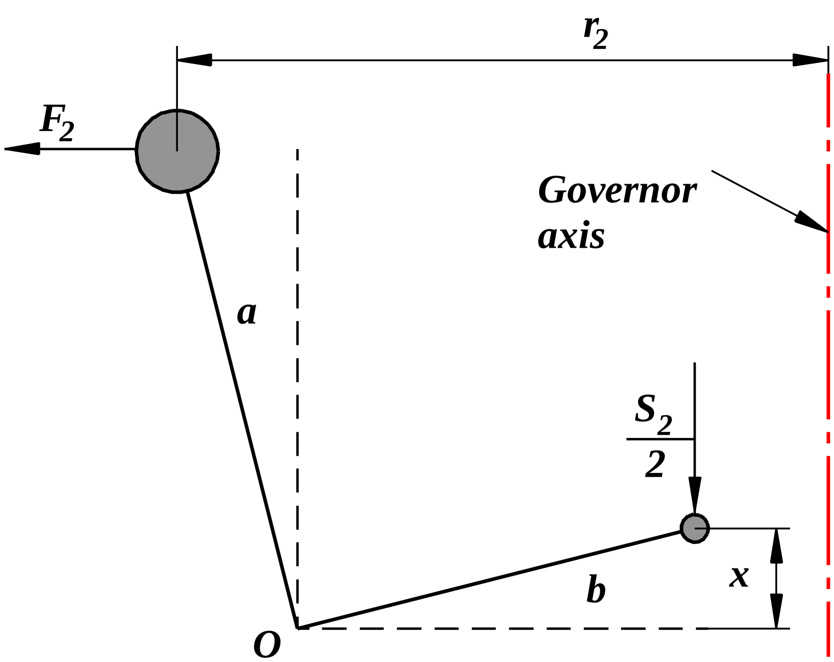
1. loads on the spring at the lowest and the highest equilibrium speeds, and
2. Stiffness of the spring.

Given : $N_1 = 290$ rpm or $\omega_1 = 2\pi \times 290/60 = 30.4$ rad/s ; $N_2 = 310$ rpm or $\omega_2 = 2\pi \times 310/60 = 32.5$ rad/s ; $x = 15$ mm $= 0.015$ m ; $a = 120$ mm $= 0.08$ m ;$b = 120$ mm $= 0.12$ m ; $r_1 = 120$ mm $= 0.12$ m ; $m = 2.5$ kg

The minimum and maximum positions of the governor are shown in Figs below.



Minimum Speed Position



Maximum Speed Position

Here,

1. ***Sleeve mass is negelcted, i.e. $M = 0$, and***
2. ***The minimum speed position is same as mean position, i.e. $r_1 = r$***

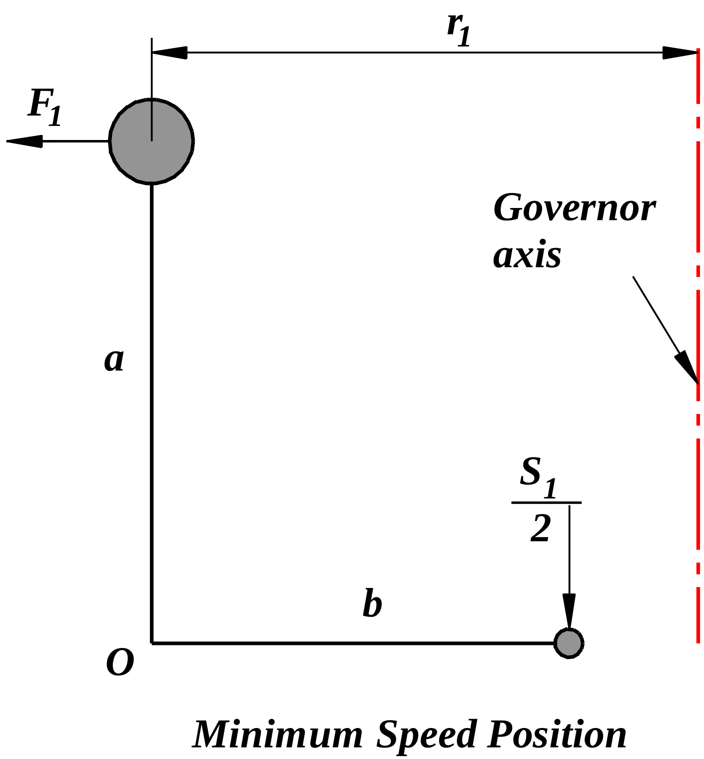


Fig (a)

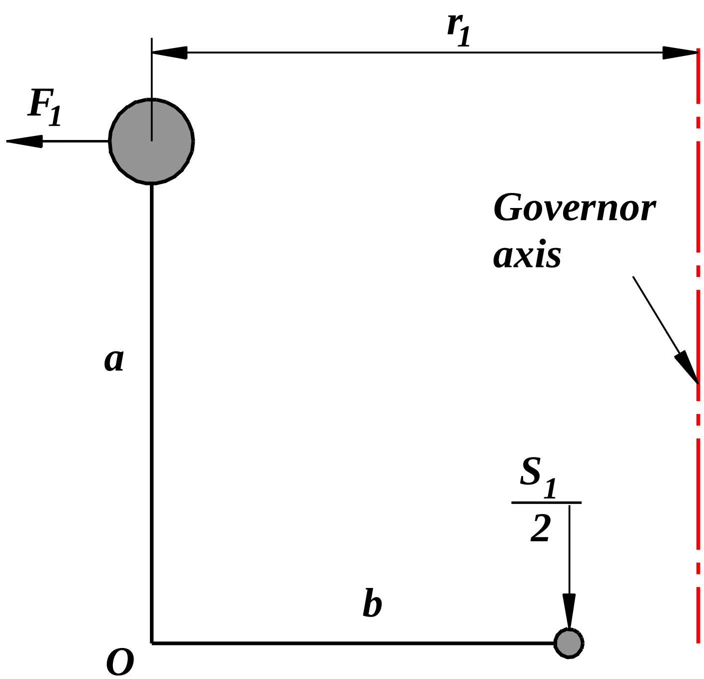
Considering the position of the ball at radius 'r_1', as shown in Figure (a) and taking moments of all forces about 'O'

$$F_1 \times a = \left(\frac{S_1}{2}\right) \times b \cdots\cdots (i)$$

where $F_1 = m\omega_1^2 r_1 = 2.5 \times 30.4^2 \times 0.12 = \mathbf{277N}$

Substituting in (i), $277 \times 0.12 = \left(\frac{S_1}{2}\right) \times 0.08$

$\therefore$ Spring force in minimum speed position $\boldsymbol{S_1 = 831N}$



Minimum Speed Position

Fig (a)

Considering the position of the ball at radius ‘r_1’, as shown in Figure (a) and taking moments of all forces about ‘O’

$$F_1 \times a = \left(\frac{S_1}{2}\right) \times b \cdots\cdots (i)$$

where $F_1 = m\omega_1^2 r_1 = 2.5 \times 30.4^2 \times 0.12 = \mathbf{277N}$

Substituting in (i), $277 \times 0.12 = \left(\frac{S_1}{2}\right) \times 0.08$

$\therefore$ Spring force in minimum speed position $\boldsymbol{S_1 = 831N}$

Properties of governors

Sensitiveness:

Consider two governors A and B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor A is more sensitive than the governor B.

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor.

The sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

$$\text{Sensitiveness} = \frac{(N_2 - N_1)}{N} \quad \text{where } N = \text{Mean speed} = \frac{N_1 + N_2}{2}$$

$$\therefore \text{Sensitiveness} = \frac{2(N_2 - N_1)}{(N_1 + N_2)}$$

Properties of governors

Stability:

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of the governor balls at which the governor is in equilibrium.

For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Note: A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Properties of governors

Isochronism:

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction.

A Porter governor can not be isochronous since the height of the governor will not be same for all radii of rotation.

However, a spring controlled governor of Hartnell type can be made isochronous by adjusting the spring tension.

Properties of governors

Hunting:

A governor is said to be hunt if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place.

For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position.

This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position.

Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely.

Properties of governors

Effort and Power of a Governor:

The **effort** of a governor is the mean force exerted at the sleeve for a given percentage change of speed.

It may be noted that when the governor is running steadily, there is no force at the sleeve. But, when the speed changes, there is a resistance at the sleeve which opposes its motion.

It is assumed that this resistance which is equal to the effort, varies uniformly from a maximum value to zero while the governor moves into its new position of equilibrium.

The **power** of a governor is the work done at the sleeve for a given percentage change of speed.

It is the product of the mean value of the effort and the distance through which the sleeve moves.

Mathematically,

Power = Mean effort × lift of sleeve

Controlling Force

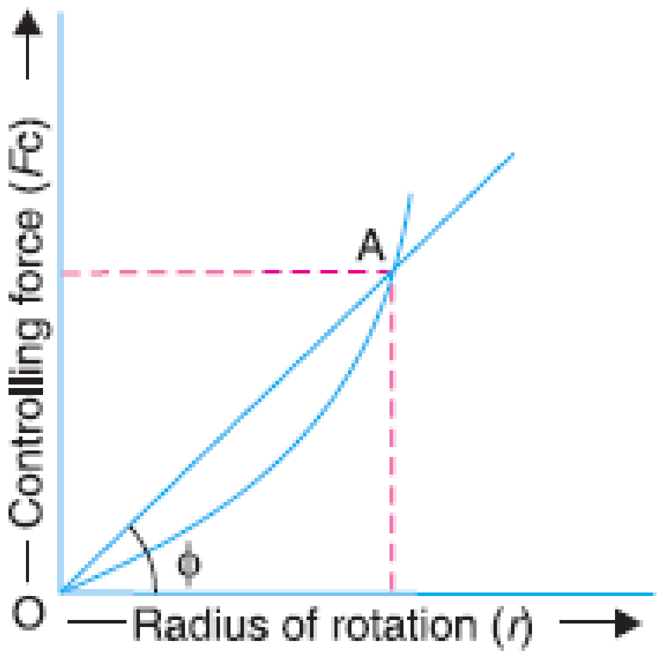
In case of a governor running at a steady speed, the inward force acting on the rotating balls is known as controlling force. It is equal and opposite to the centrifugal reaction.

∴ Controlling force, $F_C = m.\omega^2.r$

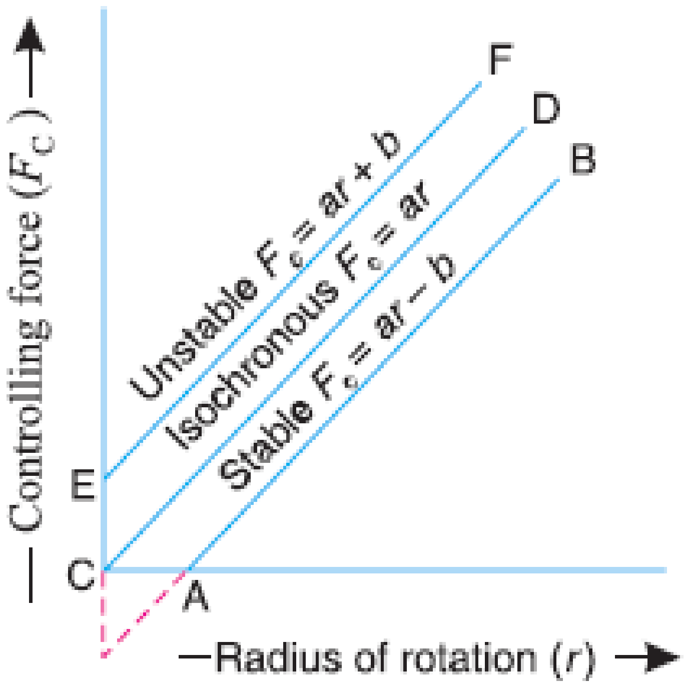
The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor).

When the graph between the controlling force (F_C) as ordinate and radius of rotation of the balls (r) as abscissa is drawn, then the graph obtained is known as ***controlling force diagram***. This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.

Controlling Force Diagram for Porter & Hartnell Governors



Controlling Force Diagram for Porter Governor



Controlling Force Diagram for Hartnell Governor

Controlling Force Diagram for Spring-controlled Governors

The controlling force diagram for the spring controlled governors is a straight line, as shown in Fig. We know that controlling force, $F_C = m.\omega^2.r$ or $F_C /r = m.\omega^2$

The following points, for the stability of spring-controlled governors, may be noted :

1. For the governor to be stable, the controlling force (F_C) must increase as the radius of rotation (r) increases, i.e. F_C / r must increase as r increases. Hence the controlling force line AB when produced must intersect the controlling force axis below the origin, as shown in Fig.

The relation between the controlling force (F_C) and the radius of rotation (r) for the stability of spring controlled governors is given by the following equation

$F_C = a.r - b$ . . . (i) where a and b are constants.

2. The value of b in equation (i) may be made either zero or positive by increasing the initial tension of the spring. If b is zero, the controlling force line CD passes through the origin and the governor becomes isochronous because F_C /r will remain constant for all radii of rotation.

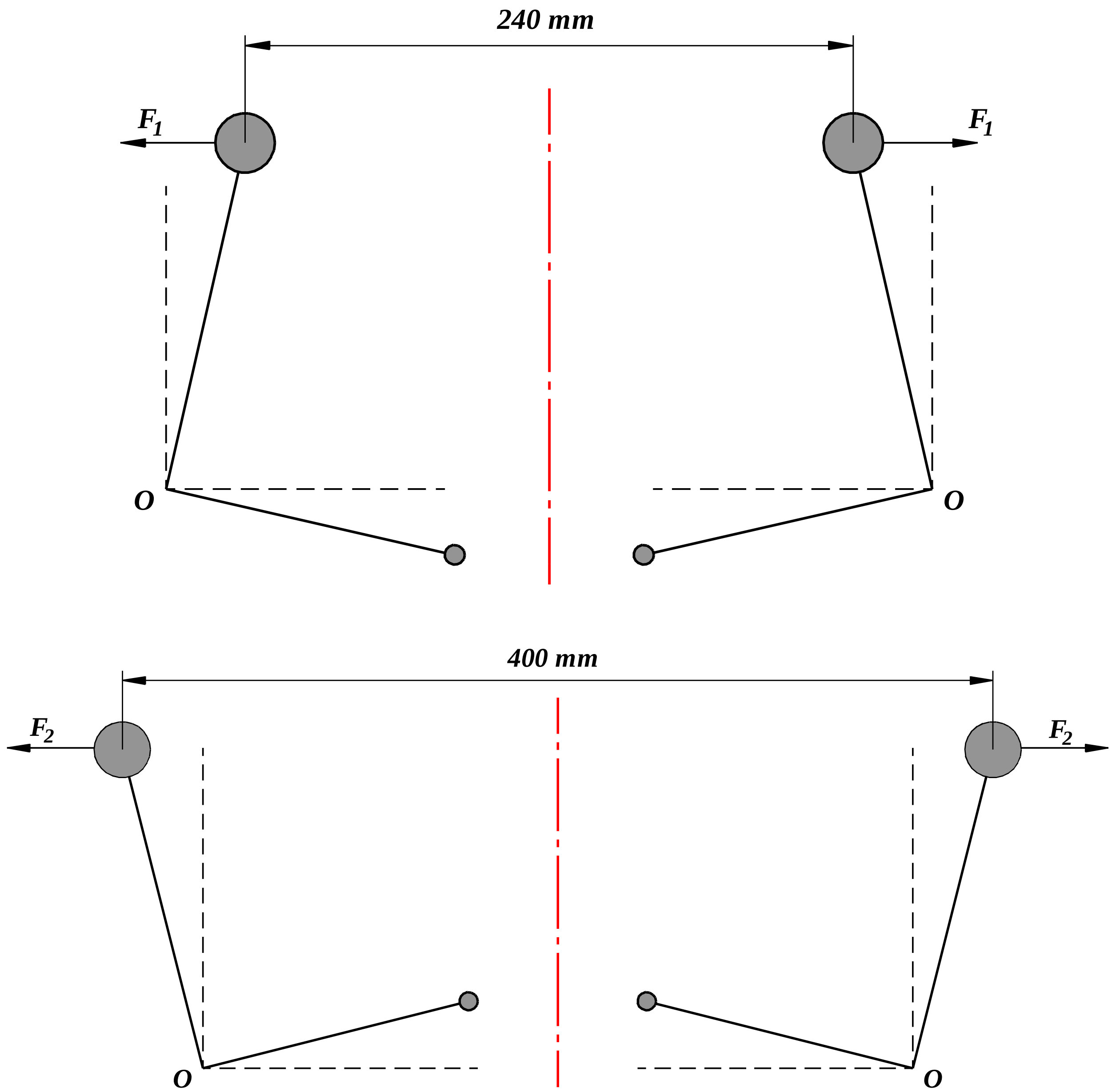
The relation between the controlling force and the radius of rotation, for an isochronous governor is, therefore, $F_C = a.r$ *. . . (ii)*

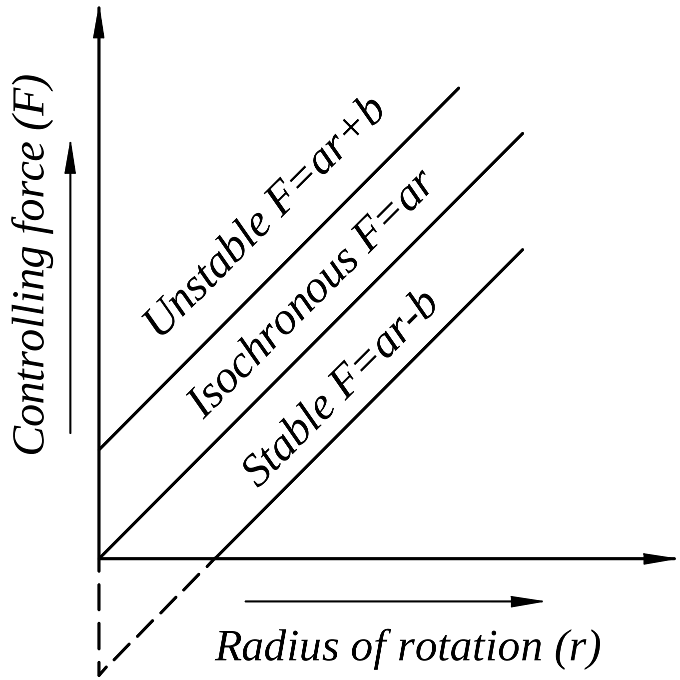
3. If b is greater than zero or positive, then F_C /r decreases as r increases, so that the equilibrium speed of the governor decreases with an increase of the radius of rotation of balls, which is impracticable. Such a governor is said to be unstable and the relation between the controlling force and the radius of

rotation is, therefore $F_C = a.r + b$

Problem 7

In a spring loaded governor, the controlling force diagram is a straight line. The balls are 400 mm apart when the controlling force is 1500 N & 240 mm when it is 800 N. The mass of each ball is 10 kg. Determine the speed at which the governor operates when the balls are 300 mm apart. By how much should the initial tension be increased to make the governor isochronous? Also find the isochronous speed.


240 mm
F1
F1
O
O
400 mm
F2
F2
O
O



The equation of a straight line is F=ar+b.

when $r = 0.2m,\ 1500 = 0.2\ a + b \cdots\cdots (i)$

when $r = 0.12m,\ 800 = 0.12\ a + b \cdots\cdots (ii)$

Solving (i) & (ii), $a = 8750\ N/m,\ b = -250$

$\therefore$ ***F = 8750r - 250*** $\Rightarrow$ It is a stable governor.

When r = 150 mm = 0.15m,

$F = (8750 \times 0.15) - 250 = 1062.5N$

When F = ar, the governor is isochronous

$m\omega^2 r = 8750 \times r$

$$\Rightarrow 10\left(\frac{2\pi \times N}{60}\right)^2 r = 8750 \times r$$

$\therefore$ ***Isochronous speed = 282.5 rpm***